THE ARITHMETIC TEACHER

Volume 8, number 3 MARCH 1961

Suggestions for teaching the concept of remainders in division, development of new curriculum materials in mathematics for the talented, and a review of books on numbers

A JOURNAL OF *The National Council of Teachers of Mathematics*

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MARCH 1961 *Volume 8, number 3*

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a journal of The National Council of Teachers of Mathematics

All editorial correspondence, including books for review, should be addressed to the Editor. All manuscripts submitted for publication should be in duplicate copy. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

THE NATIONAL COUNCIL
OF TEACHERS OF MATHEMATICS
1201 SIXTEENTH STREET, N.W.
WASHINGTON 6, D.C.

Second-class postage paid at Washington, D.C., and at additional mailing offices.

THE ARITHMETIC TEACHER is published monthly eight times a year, October through May. The individual subscription price of $5.00 ($1.50 to students) includes membership in the Council. (For an additional $3.00, or $1.00 to students, the subscriber may also receive *The Mathematics Teacher*.

Institutional subscription: $7.00 per year for each journal. Single copies: 85¢. Make remittance payable to The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

About the articles

E. W. HAMILTON *Associate Editor*

Training of space chimps to a set of conditioned reflexes, like the "education" of the "little brothers—the Proles" in Orwell's *1984*, is pretty well systematized. But how do you teach youngsters to create, to invent, or even to observe and investigate as do the comparatively small number of really productive people in the frontier areas of our culture?

Specifically, how do we who teach arithmetic help to produce some mathematicians along with the great horde of bookkeepers, public and private, which our economy demands?

We recognize occasional talented students, but how many more might emerge from the mass if given different experiences and stimulation? The task of providing these experiences is not so well understood as that of establishing conditioned reflexes. A Russian provided the classic experiment in conditioning sixty years ago. Should we wait for the Russians to lead again?

Lansdown and Hohn, who provide us with the two leading articles this month, present some philosophical discussions of the problem and some suggestions, both for teacher preparation and for classroom activity. Articles by Heard, Driscoll, and Jackson extend the scope of suggested activities in several directions and at lower-, middle-, and upper-grade levels.

The critical element in any of these suggested activities is the teacher, a mathematics-minded teacher in a situation free enough to allow pursuit of knowledge for knowledge's sake and able to enjoy both children and ideas as they develop. After you have read the first two articles, browse for your particular grade interest. Then try to imagine your own teacher in fourth grade, as described by Driscoll, exploring a general idea just to explain a few questions. Why, the children didn't even know there were such questions, to say nothing of the answers, until she led them on!

Or, try your imagination on some upper-grade classrom of five to fifty years ago. Can't you just feel the guilt building up at the waste of time on the properties of number, as described by Jackson, when the time could be spent in multiplying and dividing decimal fractions?

And, as one of the ways to provide the necessary free situation for the able teacher, the final group of three articles give reports on several aspects of the recently revived question of homogeneous grouping. McLaughlin's account of careful organization and promotion and his tabulation of results should be of help to anyone interested in starting such a program. Pinney's report displays some evidence which may help to convince the doubters that it might be worth trying, and Lerch reviews some of the literature on attitudes possessed by or developed by students subjected to homogeneous grouping.

Creating mathematicians*

Poeta fit non nascitur

BRENDA LANSDOWN *Brooklyn College, New York, New York*
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Much foundation and industrial money is spent on searching out existing talent. Perhaps a more democratic approach to conserving our national intellectual resources is to take the view that we can create talent.

Viewing creation as a projection of personality

One creates only in one's own image. The created object is in some measure a projection of one's personality. For this reason, we can always distinguish a painting by Van Gogh from one by Mondrian. To create mathematicians in the elementary, or even in the nursery school, the teacher should be a mathematician or at least mathematics-minded as it is defined later in this article. However, elementary-school teachers cannot be converted into mathematicians by the usual "methods instructors" in departments or schools of education. They need the touch of real mathematicians from the mathematics departments of liberal arts colleges.

Here the analogy of the painter must be modified. The painter begins his creating with inert, standardized materials. When a teacher sets out to create a type of person, the material has already been worked on both by heredity and environment; the personality is formed in some of its basic patterns by the age of seven according to the major schools of psychology. Does this mean that the mathematician is made or marred by the time a child enters school? Margaret Mead, in a speech to the national convention of the National Science Teachers Association in Denver, Colorado, 1958, suggested that mathematics ability can be founded in the nursery-school years. But it has never been established that there is a gene for mathematics nor that the ability to perform well in the field is finally determined before the child has learned any mathematics at all. It is the thesis of this article that many more children can be trained in mathematics-mindedness than have ever been so far; that many of these so trained can become mathematicians; that the role played (unwittingly) by countless numbers of elementary-school teachers is to crush from the potential mathematician all desire to cope with number relations chiefly because the teacher cannot cope with them. This does not say that the elementary-school teachers cannot juggle the symbols around and create children who become skilled in finding answers by the the dot-and-carry-one routine. Unfortunately, those who succeed in this kind of virtuosity tend to like arithmetic (as presented in many textbooks) and to carry on similarly in the higher grades.

Not every child can become a mathematician. Of course one excludes the brain-injured child whose raw material will forever prevent him from gaining a Nobel

* From *New York State Mathematics Teachers Journal,* June 1960.

prize in theoretical physics. Who knows, however, how many children within the average range of educability become mathophobes instead of mathophiles? This is largely an emotional problem whereby fears and experiences of frustration and failure have been attached to the arithmetic lessons and thereby to number relations in general. In large measure, this attitude is due to the teacher who cannot enter into the specialized thinking of mathematics, but considers number relations as an ogre to be placated instead of controlled.

Mathematics and personality

Is there a special personality needed for the career of mathematician? The literature, although sparse, would indicate that there is. A summary of the researches in this area by Super and Bachrach[1] suggests:

> The personality structure of the mathematician seems to be that of an objective, cold, and competitive individual in his interpersonal relations, with relative freedom from anxiety and little affect. At the same time the mathematician is normal in his social contacts, he is not an eccentric.

On the other hand, in a long-range study at Brooklyn College, sparked by Professor Carleton Washburne and conducted by Professor Louis M. Heil, it is becoming evident that the student who is successful in his mathematics studies is a person characterized by turbulence both in feelings and thought. He can tolerate and even prefers disorder. He accepts his own impulses, expresses his aggression verbally and is nonaccepting of authority figures. Such a person, whom we have called the A-type, does not produce the best results as a teacher for most types of young children. The best elementary-school-teacher personality, according to the same study, is that of a person who has a great deal of self-control, a strong achievement drive (which is communicated to the children), who accepts children's feelings while repressing his own. He is low in verbal aggression, likes to have things well organized and running smoothly, and is somewhat submissive to authority figures. We have called this personality the B-type.[2] The B-type teacher not only gets the highest achievement from his children (when ethnic factors and socio-economic class are controlled), but reduces anxiety in his pupils and creates an atmosphere where the children tend to accept each other. It seems that all types of children learn well under the B-type teacher, and we must assume that the learning in mathematics is no exception. The problem then becomes: what kind of mathematics will be taught?

The dilemma—who should teach mathematics?

If it takes a mathematician to create a mathematician, which is our assumption, but the personality of the mathematician is not that which produces the best learning climate for elementary-school children, what then can be done? At Brooklyn College, we think we have found the bridge, the bridge between the mathematicians' special subject area and the personality which flourishes in it, and the psychology of the elementary-school child. We do not need a mathematician to create embryo mathematicians, but we need a teacher who is mathematics-minded. We hold that mathematics-mindedness can be trained or fostered in any type of personality. Hence we can train the elementary-school teacher to think mathematically, and he can pass on this ability to the various types of children in his classes. The B-type teacher will pass it on best.

What mathematics-mindedness means

Mathematics-mindedness, as we see it, is, for one thing, the ability to have mathematics concepts under one's control, to be able to apply concepts in various situations, to be able to envisage possible answers to problems without having to do all the calculations, to understand the

axiom-postulate system upon which mathematics is built. As an example of the lack of mathematics-mindedness we can cite our average education major who conceives multiplying by ten as adding a zero, leaving a space, putting down a dot, or going through the routine of applying the tens' table. We should prefer a student who conceives of multiplying by the base number in any number system as endowing each numeral with the value it would have if it were moved one place to the left. We would like to have our education graduates feel at home in three-dimensional space concepts. They should be able to imagine the plane of the elliptical orbit of a man-made satellite cutting the earth in a great circle, instead of confining themselves to the homaloidal world of pencil mark and paper.

Such outcomes of conceptual thinking might be expected from students who have taken well-planned courses in a college mathematics department. This, however, is not the case for many nonmathematics majors. A student coming through the nonmathematics-minded teaching of the lower grades is shattered by the thinking of the college mathematics teacher. He is left far behind the finer or most pertinent offerings of the course, whereas the real mathematician may follow along joyously.

One solution: a special course for elementary-school teachers

What is needed, we decided, after six months of committee meetings by representatives from both the departments of education and mathematics, is a special noncredit course for elementary-education majors, given by the mathematics department but geared to the outcomes presented by the education department. The course will be heavy on concepts, and light on higher-level problem-solving. It will teach basic mathematics principles, the axiom-postulate system, and help the student apply these to various fields. It will present the ideas embodied in the theory of sets and in the principles of variously based number systems. It will help the student approach a problem from a Gestalt viewpoint, to recenter his perception instead of wading through oceans of calculations which may not lead to a solution because of technical errors. Elementary-school teachers have found considerable inspiration from the Gestalt approach of Max Wertheimer's little book called *Productive Thinking*.[3] Can mathematicians convey these ideas to the B-type personality? We believe that on the adult level they can.

To find out which of our students at Brooklyn College need such a special course we have prepared and administered a "Mathematics Competency Test"[4] in which the questions are so phrased that they cannot be answered by an algorism reflex. For instance, we use bead-frame diagrams to probe concepts of multiplying in number systems based on two and five. For the many students at present who cannot think in these terms, the course mentioned above is being prepared. For students needing less help, but who still need to improve their thinking in the field of mathematics as revealed by the Competency Test, we have suggested that they study on their own one of two recent texts designed for this purpose: Robert L. Swain, *Understanding Arithmetic* (New York: Rinehart, 1957); William L. Schaaf, *Basic Concepts of Elementary Mathematics* (New York: Wiley, 1960).

Some students, we feel, will not be able to cope even with these clear and stimulating books entirely on their own, so we have arranged that mathematics tutors in our Basic Skills Center will be available for short-term help with small groups of students. This will serve also the excellent purpose of bringing into face-to-face relationship the profound puzzlement and unclear thinking of the future elementary-school teacher with an expert in the field of mathematics, a person who, hitherto, concerned himself mainly with communicating with mathematicians. We

have assumed throughout that persons who are creative mathematicians in their own right can find ways to transfer mathematics-mindedness to young adults who certainly will never become mathematicians themselves.

Summary

In this article we have separated the qualities which belong only to the creative mathematician from those we call "mathematics-mindedness." We have mustered evidence that the mathematician has personality characteristics different from those of the good elementary-school teacher. We have assumed that only mathematicians can create mathematicians, but that mathematics-mindedness is a literacy heritage which can be taught to any child by teachers who are themselves trained to be mathematics-minded. We believe that mathematicians who are willing to accept the task are the best people to train mathematics-mindedness in young adults of various personality types. If we do not pursue these aims, we are likely to lose our potential mathematicians because they have not been readied in mathematics-mindedness by their early schooling—they become mathophobes long before they have the good fortune to meet creative mathematicians in college.

Notes

1. Donald E. Super, Paul B. Bachrach, *Scientific Careers and Vocational Development Theory* (New York: Bureau of Publications, Teachers College, Columbia University, 1957), p. 53.
2. There are other types of personalities to be found among teachers in our school systems, but these produce notably poorer results than either the A-type or the B-type of personality.
3. Max Wertheimer, *Productive Thinking* (New York: Harper Brothers, rev. ed., 1957).
4. Prepared by the author of this article.

Ten weary, footsore travelers,
All in a woeful plight,
Sought shelter at a wayside inn
One dark and stormy night.

"Nine rooms, no more," the landlord said,
"Have I to offer you.
To each of eight a single bed,
But the ninth must serve for two."

A din arose. The troubled host
Could only scratch his head,
For of those tired men no two
Would occupy one bed.

The puzzled host was soon at ease—
He was a clever man—
And so to please his guests devised
This most ingenious plan.

In room marked A two men were placed,
The third was lodged in B,
The fourth to C was then assigned,
The fifth retired to D.

In E the sixth he tucked away,
In F the seventh man,
The eighth and ninth in G and H,
And then to A he ran,
Wherein the host, as I have said,
Had laid two travelers by;
Then taking one—the tenth and last—
He lodged him safe in I.

Nine single rooms—a room for each—
Were made to serve for ten;
And this it is that puzzles me
And many wiser men.

—MARTIN GARDNER, *Mathematical Puzzles and Diversions* (New York: Simon and Schuster, 1959) pp. 142–143.

If we reflect on what he's done,
We'll see we're not insane.
Two men in A, he's counted one,
Not once, but once again.

Op. cit.
—Submitted by JOHN F. MOONEY, *Ebasco International Corporation*, New York, New York, p. 149.

Teaching creativity in mathematics*

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In addition to his teaching and research responsibilities as associate professor of mathematics in the Department of Mathematics, Dr. Hohn has been serving as a consultant and writer for the University of Illinois Arithmetic Project.

According to their own declaration, many—perhaps even most—children do not like mathematics. In the case of bright children, children with creative minds, this results in a tragic loss, not only to society in technical power, but also to the individual in ability to appreciate a subject of great interest and beauty.

Computation and mathematics

Most of those who do not like mathematics, and probably even many of those who do like it, do not know what mathematics really is. Most people, including too many teachers, confuse mathematics with a body of manipulative skills which must be learned to the point of automatic response, in order that an individual may function successfully and usefully in certain important circumstances. Though many mathematicians possess these skills to a high degree, the skills themselves do not characterize the mathematician, nor does exercise of them constitute his principal activity.

The true mathematician, as contrasted with the expert in computation, is uninspired by the prospect of performing routine, repetitive tasks, just as a true artist would not wish to paint many copies of the same picture. On completing one painting, the artist will turn to a new one, a new expression of the beauty or of the concerns he finds within himself. In a similar way, the mathematician's primary creative activity is the study of patterns, relationships, forms, and structures in systems of numbers, geometrical figures, functions, and other objects of interest. Just as we want the child to learn something of music, art, and literature, so that he may appreciate to an optimum degree these accomplishments of the human mind and spirit even though he may not become a musician, an artist, or a poet, so also should he learn something of the real spirit of mathematics, which is one of the most elegant creations of the human mind.

The mathematician's creative task

It is important to realize that the mathematician's work includes much more than the study of relationships in the worlds of numbers and geometrical figures. He is free to look for form and structure wherever he can find them, for example in such fields as biology, physics, chemistry, agriculture, psychology, and many others. Often he will have to invent new symbolic tools for describing what he sees. In many cases, the farther one goes from the world of numbers, the more difficult it becomes to detect and describe mathematically the patterns that appear. But in every scientific field, the detection of mathematical patterns is becoming increasingly important because the understanding so obtained promises prediction and control. Thus every scientist is potentially a crea-

* This article is also to appear in the Spring, 1961 issue of *Illinois School Board Journal.*

tive mathematician to some degree. Hence every child whose gifts equip him for possible future scientific work should learn the tools which can make his work inspiring rather than routine, creative rather than imitative.

Can children use mathematics creatively?

Now what does all this imply regarding the nature of the mathematical training we give our children, especially the more able ones? Does it mean that we must do a superb job of teaching the computational and manipulative skills and do it as early as possible? Indeed it does, but it means much more. It means that we must teach even the small child to think creatively whenever mathematical problems are to be solved. This should ultimately be the purpose of every mathematics teacher. It is of course the antithesis of the style of teaching that says, "Here is the rule. Now follow it exactly in the following list of problems." This latter style of teaching has effectively taught most of us adults not to think when we are faced with a mathematical problem, unless there is no way to avoid thinking. And then, faced with the need for real thought, we usually give up in despair, for in school we were not taught, perhaps not even allowed, to tackle problems creatively.

In contrast to the classical emphasis on following the rule, experimenters with new materials have demonstrated that a creative approach to problem solving can be used at all ages to introduce significant mathematical concepts and methods of thought. To help in this, special visual and manual devices can be employed in such a way as to teach the child to invent and test alternative approaches, and to grasp intuitively and almost instantly relationships that could be expressed only with the greatest of difficulty in words that the child would understand. Such devices can be used as vehicles for creative rather than uninspired drill in basic skills and thus help to speed up the perfection of these skills by providing proper motivation. Indeed, repeated experience shows that such inspiration often makes an accomplished student of a youngster who is accused of being dull but is in fact only psychologically unable to memorize mathematical facts efficiently under duress.

Devices such as those referred to can also be used to show that not all mathematical problems need to follow the familiar patterns of the number system, as well as that the number system itself displays exciting and instructive patterns most people never see and don't even know how to see. The children should also learn that not all problems have ready solutions, and that some problems may have, at least in terms of resources momentarily available, no solution at all.

Of particular importance is the use of a wide variety of problems and techniques to prevent development of the misunderstanding that everything goes according to a few familiar patterns and to develop a sense of complete freedom in the generating of creative ideas. The object is to develop in the children minds free from the shackles of routine thinking, to release for a lifetime the powers of a creative mind.

What happens when the approach is creative?

The teacher who leads children into discovery must always be prepared for shock. Often he will be in a position like that of a parent taking a child for a walk in the woods. "What's *that* thing?" "Look over there!" "What's making that funny noise?" And the parent, trained by long practice (as we all are) *not* to see and *not* to hear, is ashamed to realize how much he has been missing. In the same way, children often see relationships and significances that the teacher and even the professional mathematician, hampered by conventional ways of looking at things, might miss.

Another shock comes when the children ask questions the teacher cannot answer, pose problems the teacher cannot solve. In the discovery method, it is absolutely essential that this situation be accepted as normal and indeed as desirable rather than being considered a cause for distress. Children are reasonable: they *won't* expect you to know everything but they *will* delight in helping you work out the answers. Moreover, some of the finest creative teaching comes out of precisely such a mutual searching for solutions to problems that children have raised.

Another source of distress may be the children's guessing. (Perhaps adults don't guess: they simply "make conjectures.") To me, the most exciting moments in an elementary classroom have been those when the children, faced by a difficult problem, began making wild and beautiful guesses. How on earth could they ever think of the things they do think of? This behavior is close to that of the research mathematician when he tackles an unfamiliar problem. Let the children guess and learn to test their guesses, for such is the road to discovery and to new problems to be solved.

To the teacher who appreciates both children and mathematics, such moments of discovery in the classroom provide a supreme thrill of creative accomplishment that only those who have experienced it can fully understand. But be forewarned! Do not experiment too casually with the creative approach, for once you have tasted success, you will be forever dissatisfied with conventional materials and with your own attainments. Indeed, such dissatisfaction is one mark of the true teacher. Its reward is a growing sense of accomplishment arising out of the knowledge that there is no more significant privilege than to release the creative powers of a child's mind.

Classroom situations

The object of the examples which follow is to show a few of the ways in which the above-stated purposes can be accomplished. Materials such as these can be used to give the child experience in all the essential aspects of mathematical research: the formulation, testing, acceptance, or rejection and reformulation of hypotheses until the correct conclusion is reached. To give children such experience, which they find truly exciting, one must lead them to discover the basic aspects of a new situation for themselves rather than hand them a previously prepared conclusion. Only in this way can they learn to develop insight rather than to depend on thinking in habitual patterns firmly established by drill. Experience in discovery, in a wide range of situations, is the critical factor in the development of genuine mathematical thinking.

Example 1

By now most everyone has seen the symbolism of "frames." For example,

$$\square \rightarrow \square + 3.$$

This is read "Box goes into box plus three" and requires that whatever number is put into the first box must also be put into the second box. For example,

$$\boxed{5} \rightarrow \boxed{5} + 3 \qquad \boxed{9} \rightarrow \boxed{9} + 3$$

$$\text{i.e.,} \quad 5 \rightarrow 5+3=8 \qquad 9 \rightarrow 9+3=12.$$

Interpreted geometrically on a number line, the "game"

$$\square \rightarrow \square + 3$$

involves shifting every point three units to the right. (The children should tell you this of course.) In addition to introducing informally the basic concept of a kind of transformation known as a "translation," such a game gives opportunity for much interesting drill in addition, even as far down as the first grade.

Now you vary the game a little:

$$\square \rightarrow \square - 4.$$

This is fine until you try to insert numbers less than 4. Then what should you do? I

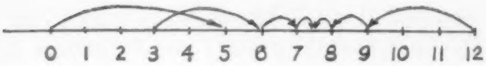

Figure 1

have seen first graders discover for themselves in this way the necessity for inventing negative numbers and discover intuitively the rules for adding and subtracting signed numbers. It is nonsense to postpone the discovery of negative numbers until high school when they are so easy and so much fun to treat earlier.

Example 2
Consider the number-line game

$$\square \rightarrow \frac{\square}{3}+5.$$

Here

$$9 \rightarrow \frac{9}{3}+5=8$$

$$8 \rightarrow \frac{8}{3}+5=7\tfrac{2}{3}$$

$$7 \rightarrow \frac{7}{3}+5=7\tfrac{1}{3}$$

$$6 \rightarrow \frac{6}{3}+5=7$$

Let us show this on a figure like that in Figure 1.

Is there any point which behaves in a peculiar way? The children will soon point out that

$$7\tfrac{1}{2} \rightarrow \frac{7\frac{1}{2}}{3}+5=2\tfrac{1}{2}+5=7\tfrac{1}{2}.$$

Here is good practice in fractions as well as an intuitive introduction to the important mathematical concept of a fixed point: a point which is invariant under a given transformation. (Notice how little is conveyed by the preceding clause, but how clearly the same concept is conveyed by the example.)

In one class I saw this happen: Two parallel number lines were drawn, as shown in Figure 2. The procedure now was to connect corresponding points with lines. E.g., since

$$12 \rightarrow \frac{12}{3}+5=4+5=9,$$

connect 12 on the lower line with 9 on the upper line. This figure, after the connecting lines are extended, gives insight into the transformation called "projection." It was a girl student in the sixth grade who proposed this extension of the lines and remarked, "The point where the lines in-

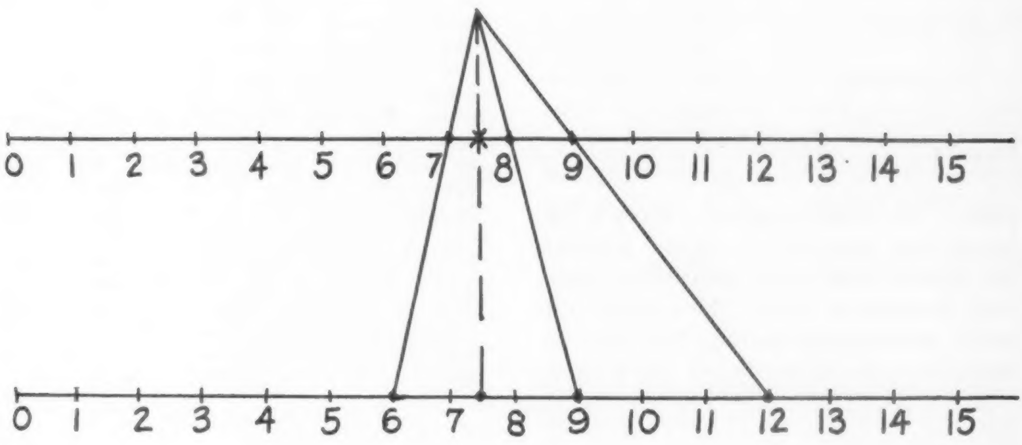

Figure 2

tersect tells you the stand-still point." Indeed, the point of intersection is the center of a projection which connects the point named $7\frac{1}{2}$ on one line to the point named $7\frac{1}{2}$ on the other line.

Example 3

One student looked at a table of integers to the base 7:

0	1	2	3	4	5	6
10	11	12	13	14	15	16
20	21	22	23	24	25	26
30	31	32	33	34	35	36
40	41	42	43	44	45	46
50	51	52	53	54	55	56
60	61	62	63	64	65	66

He remarked, "In base 7, our ordinary 8 becomes 11, 10 becomes 13, and so on. These are even numbers, but they don't look even. How do you tell an even number in base 7? Oh, I know!" What rule had he discovered? How does one identify even numbers in an arbitrary odd base? Can you prove the correctness of your rule by using the place-value concept?

Example 4

A teacher asked her fourth grade to bring in patterns that could be folded up to make a cube. Several different patterns came in, some like those in Figure 3, and

Figure 3

others. One child objected, "Hers is different from mine. It isn't right." A simple test showed both were right. Then somebody wanted to know, "How many different patterns are there?" The next assignment was, obviously, "I don't know, so *you* find out." The children found "all" the patterns. Why don't you find them all and prove that your answer is right?

Of course, finding all the patterns isn't possible unless the problem is properly defined, so a discussion of what is to be acceptable as a "pattern" is necessary. What, in fact, is a precise definition of the problem? Can you prove very simply that there exists only a finite number of solutions to the problem you have defined? How many solutions are there if the cube is replaced by a rectangular box with only two equal dimensions? With no equal dimensions? What differences are there between a mathematical solution and a physical solution to the problem?

Problems such as these can be found in many places, but you don't have to look far for them if the classroom atmosphere is right: they arise naturally.* And you don't have to be able to see the full significance of a problem at once. The children will help you discover the significance if you let them, and everybody will learn from the process.

* The above examples are typical of experiences in classes conducted by the University of Illinois Arithmetic Project, 1207 West Stoughton, Urbana, Illinois. Further information about the project may be obtained by writing to Professor David Page, Director, at the above address.

The *Symbols of Addition and Subtraction*, (+) and (−), were first introduced by Michael Stifel, a German mathematician of the sixteenth century. They first appeared in a work published by him at Nuremberg, in 1544, and are believed to have been invented by him. This is implied by the manner in which he introduces them: "thus, we place this sign," etc., and "we say that the addition is thus completed," etc. Prof. Rigaud supposed that + was a corruption of *P*, the initial of *plus*, and Dr. Davis thought that it was a corruption of *et* or &. Stifel, however, does not call the signs *plus* and *minus*, but *signum additorum* and *signum subtractorum*, which renders these suppositions improbable. —EDWARD BROOKS, *The Philosophy of Arithmetic* (Lancaster: Normal Publishing Co., 1880), p. 109.

Creative thinking and discovery

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During the past year the writer was privileged to be a member for the second year of the Michigan Center of the *School Mathematics Study Group.* One of the classes in which the seventh-grade SMSG material was taught was an accelerated group in which the range of I.Q.'s was from 118 to 170. The students of this group were classified as gifted.

Enthusiasm and interest were high as the members of the class realized that they were part of the special nationwide program to improve the content of arithmetic for the seventh grade.

Many fine, constructive suggestions were made by these students, showing evidence of creative thinking and imagination. One student wrote an original poem about "Zero."[1] Another example of creative thinking is the following suggestion, which was offered during the study of the unit on Factors and Primes.

Exploring problems that involve complete factorization

Consider the complete factorization of two or more numbers—let us take the set of prime factors of the counting number 30 and call this Set A.

$$\text{Set } A=\{2, 3, 5\}.$$

Set B will consist of the set of prime factors for the counting number 21.

$$\text{Set } B=\{3, 7\}.$$

[1] The Arithmetic Teacher, VII (March, 1960), 160.

Then the intersection of Set A and Set B would be

$$A\cap B=\{3\}.$$

It can be observed that this intersection is the *Greatest Common Factor* for the counting numbers 30 and 21.

The union of these two sets of prime factors (30 and 21), is the set of factors which when multiplied together gives 210, the *Least Common Multiple* of 30 and 21.

$$A\cup B=\{2, 3, 5, 7\}.$$

The idea of using intersection and union of sets of prime factors to find the Greatest Common Factor and the Least Common Multiple was discovered by a student of this gifted class. This idea was taken to the Michigan Center for discussion and later modified to include the proper designation for indicating the identity of two or more factors of the same value by the use of subscripts.

The problem of representing sets of prime factors, when there is more than one of the same value, can be seen in the following example. Let us take the sets of prime factors of the counting numbers 12 and 20.

Let C represent the set of prime factors of 12,

$$\text{Set } C=\{2_1, 2_2, 3\},$$

and let D represent the set of prime factors of 20.

$$\text{Set } D=\{2_1, 2_2, 5\}.$$

Then the Greatest Common Factor of 12 and 20 can be shown by the intersection of these two sets,

$$C \cap D = \{2_1, 2_2\}.$$

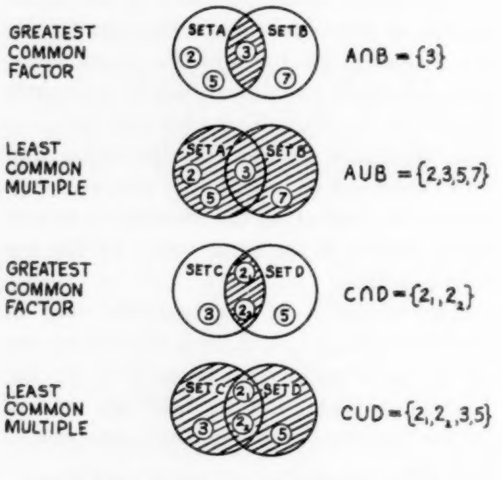
Note: The Greatest Common Factor is found by multiplying the elements of the intersection, 2×2 which equals 4, and the Least Common Multiple of 12 and 20 is found to be the product of the factors of the union of these two sets,

$$CD = \{2_1, 2_2, 3, 5\}.$$

Note: The Least Common Multiple of 12 and 20 is found by multiplying the factors $2 \times 2 \times 3 \times 5$ which is 60.

Later in the year the class was quick to realize the possibilities of representing these relationships in Venn Diagram form. The following illustrations show how these values appear in Venn Diagram form.

VENN DIAGRAMS



NUMBERS IN SMALL CIRCLES FACTORS AS ELEMENTS OF THE SETS.

While studying the unit on *Whole Numbers* the class was intrigued by the concept of *one-to-one correspondence* between sets of (*a*) counting numbers and even numbers, (*b*) counting numbers and odd numbers, (*c*) counting numbers and prime numbers, and (*d*) counting numbers and negative numbers. The fascinating ideas suggested by the fact that there were just as many even numbers, odd numbers, prime numbers, and negative numbers as there were counting numbers seemed to impress the class.

Finding new aspects of addition

The Associative Property of Addition was applied to building tens. Example:

$$\begin{aligned} 8+9 &= 8+(2+7) \\ &= (8+2)+7 \\ &= 10+7 \quad \text{or} \quad 17. \end{aligned}$$

As applied to the addition of signed numbers the class was able to show examples such as,

$$\begin{aligned} (+9)+(-13) &= (+9)+[(-9)+(-4)] \\ &= [(+9)+(-9)]+(-4) \\ &= 0+(-4) \quad \text{or} \quad -4. \end{aligned}$$

In solving equations this property was applied as well as the *additive inverse* as shown below:

$$2x+5=9$$
$$2x+(+5)+(-5)=(4+5)+(-5)$$
$$2x+[(+5)+(-5)]=4+[(+5)+(-5)]$$
$$2x+0=4+0$$
$$x=2.$$

Obtaining all the sets of two factors for a given number

The Associative Property of Multiplication, Closure, and the *Commutative Property of Multiplication* was applied when finding possible sets of 2 factors for numbers. For example: Find all possible sets of 2 factors each for the number 210.

The first set of factors is easily discovered to be 1 and 210.

The student then found the complete factorization of 210 to be the set of four factors 2, 3, 5, and 7.

By grouping the first three of these factors a second pair of factors of 210 is found to be $(2\times3\times5)\times7$ or 30×7.

A third pair of factors of 210 is easily found by grouping these factors as follows: $2\times(3\times5\times7)$ or 2×105.

By pairing the first two of these factors and the second pair, the fourth pair of factors is discovered: $(2\times3)\times(5\times7)$ or 6×35.

If we wish to pass from the factorization $(2\times3)\times(5\times7)$ to $(2\times5)\times(3\times7)$, we need to consider the property of closure and the commutative property of multiplication as well as the associative property.

Since multiplication is closed, (2×3) is a symbol for a single real number. Using the associative pattern $a\times(b\times c) = (a\times b)\times c$ with $a=(2\times3)$, $b=5$, and $c=7$ we obtain $[(2\times3)\times5]\times7$.

If we now concentrate on the expression in the bracket and once more apply the associative law of multiplication, $(a\times b)\times c=a\times(b\times c)$ with $a=2$, $b=3$, $c=5$, we obtain $[2\times(3\times5)]\times7$.

If we now apply the commutative law of multiplication, $a\times b=b\times a$, to the expression (3×5) we have $[2\times(5\times3)]\times7$.

Reassociating the three quantities inside the bracket this becomes $[(2\times5)\times3]\times7$.

Because of the closure of multiplication, (2×5) is a symbol for a single number. Invoking the associative pattern $(a\times b)\times c=a\times(b\times c)$ with $a=(2\times5)$, $b=3$ and $c=7$ we finally have $(2\times5)\times(3\times7)$, thus finding the fifth pair of factors of 210 to be 10×21.

In a like manner, by applying the same properties we discover three other pairs of factors of 210 to be $(2\times7)\times(3\times5)$ or 14×15; $3\times(2\times5\times7)$ or 3×70; and $5\times(2\times3\times7)$ or 5×42. Thus we have discovered eight different pairs of factors of 210.

Seeing useful applications

The Commutative Property of Addition was discovered to be the explanation for checking column addition by reversing the order of adding. *The Commutative Property of Multiplication* was found to be the check by reversing the order of the factors to obtain the same product. Gifted students seem to like the technical, more scientific names for these fundamental laws of mathematics. Most gifted students are vocabulary conscious and large technical words appeal to them for they like to use them—it somehow seems to identify with superior ability.

The Distributive Property of Multiplication over Addition was illustrated by the development of the perimeter formula for the rectangle. The students discovered that since

$$P=l+w+l+w,$$

then

$$P=2l+2w$$

and

$$P=2(l+w).$$

Applied to *Rational Numbers*, the Distributive Property was used to show that

$$\frac{2}{3}+\frac{4}{3}=\left(2\cdot\frac{1}{3}\right)+\left(4\cdot\frac{1}{3}\right) \text{ or } (2+4)\cdot\frac{1}{3}.$$

More of the problems that appeal to gifted students

The *Closure Property under Addition* was discovered to have appeal to the gifted students. The following definition was used to solve problems such as those listed below: "*If the sum of any two elements of a set is an element of the set, the set is closed under addition.*" Problems: (1) Is the set of even numbers closed under addition? (2) Is the set of odd numbers closed under addition? (3) Is the set of all numbers that are multiples of 5 closed under addition? (4) Is the set of prime numbers closed under addition?

The *Closure Property under Multiplication* was defined as follows: "*If the product of any two elements of a set is an element of the set, the set is closed under multiplica-*

tion." Problems such as the following were then considered: (1) Is the set of even numbers closed under multiplication? (2) Is the set of odd numbers closed under multiplication? (3) Is the set of all counting numbers having a one in the one's place closed under multiplication?

New symbols appeal to the gifted student. Some of the symbols introduced in the seventh-grade SMSG materials are listed below:

$>$ is greater than
$<$ is less than
$\neq$ is not equal to
$\cdot$ the dot for multiplication
$\cap$ intersection
$\cup$ union
$\overleftrightarrow{AB}$ line, (read, line AB)
$\overrightarrow{AB}$ ray, (read, ray AB)
$\overline{AB}$ segment, (read, segment AB)
$\approx$ is approximately equal to

Work with identity elements leads to new insights

One of the most fascinating ideas introduced in the unit on "Whole Numbers" was that of the *identity elements.* The student was led to discover these by answering the question, "Is there a number such that when it is added to any counting number the sum is the counting number?"

$$a+n=a.$$

Having built the addition table,[2] the student was able to observe that this identity element for addition was zero. This identity element was used later (1) when studying signed numbers, and (2) in solving equations.

The *Identity Element for Multiplication* was discovered in the same way, that is, by finding a number such that when it is multiplied by any counting number, the product is the counting number.

$$a \cdot n=a.$$

[2] Humphrey C. Jackson, "*Tables and Structures,*" THE ARITHMETIC TEACHER, VII (February, 1960), 71.

By building the multiplication table,[3] it was easy to see that 1 is the identity element for multiplication.

Applications of the identity element for multiplication are frequently found when studying rational numbers. The following examples are suggested:

1. Simplifying a fraction.

$$\frac{10}{14}=\frac{2\times5}{2\times7}=\frac{2}{2}\cdot\frac{5}{7} \quad \text{or} \quad \frac{5}{7}\,.$$

Note: The identity element here is the fraction

$$\frac{2}{2}\,.$$

2. Changing a division of decimals problem so that the divisor is a whole number. $3.75 \div 1.6 = n$

$$\frac{3.75}{1.6}\cdot\frac{10}{10}=\frac{3.75\times10}{1.6\times10}=\frac{37.5}{16}\,.$$

Note: The identity element here is the fraction

$$\frac{10}{10}\,.$$

3. Changing the form of a fraction when adding:

$$\frac{3}{5}+\frac{1}{4}\,.$$

In changing the fractions to denominators of 20 the student thinks,

$$\frac{3}{5}=\frac{n}{20}\,, \qquad \frac{3}{5}\cdot\frac{4}{4}=\frac{3\times4}{5\times4}=\frac{12}{20}\,.$$

$$\frac{1}{4}=\frac{n}{20}\,, \qquad \frac{1}{4}\cdot\frac{5}{5}=\frac{1\times5}{4\times5}=\frac{5}{20}\,.$$

Note: The identity elements here are the fractions

$$\frac{4}{4} \quad \text{and} \quad \frac{5}{5}\,.$$

[3] *Ibid.*, p. 72.

4. Multiplication of fractions.

$$1\tfrac{4}{5}\times 3\tfrac{1}{3}=\frac{9}{5}\times\frac{10}{3}$$

$$=\frac{3\times 3}{5}\cdot\frac{2\times 5}{3}\quad\text{(factoring)}$$

$$=\frac{3\times 3\times 2\times 5}{5\times 3}$$

$$=\frac{3}{3}\cdot\frac{5}{5}\cdot\frac{2}{1}\cdot\frac{3}{1}\quad\text{(regrouping)}$$

$$=1\cdot 1\cdot 2\cdot 3$$

$$=6$$

Note: The identity elements here are the fractions

$$\frac{3}{3}\ \text{and}\ \frac{5}{5}\,.$$

It is realized that there is no need to use the word "cancel" in any of this work, which is in agreement with the thinking of professional mathematicians and educators.

5. Division of a fraction by a fraction.

$$\frac{7}{2}\div\frac{2}{3}=\frac{\frac{7}{2}}{\frac{2}{3}}\cdot\frac{6}{6}=\frac{\frac{42}{2}}{\frac{12}{3}}=\frac{21}{4}\quad\text{or}\quad 5\tfrac{1}{4}.$$

Note: The identity element occurs here as the fraction

$$\frac{6}{6}\,.$$

In the unit on Whole Numbers the student discovers the meaning of *inverse* processes, *betweenness*, and is helped to realize the many special properties of the number *one* and the number *zero*.

The values of stimulating creative thinking and discovery

It has been the purpose of this article to acquaint teachers of arithmetic with some of the ideas advocated by the School Mathematics Study Group as presented in the seventh-grade material and to show how this material stimulates creative thinking and discovery. It is thought by many that most of these ideas could be introduced earlier in the students' experiences, perhaps in the later elementary grades. This would probably be true if one had gifted students.

Teachers who have had the opportunity to experiment along this line find that gifted students are stimulated by new ideas, by rational explanations of the arithmetic processes, and by discovery techniques. Not only do they readily assimilate new ideas, but they are led through discovery to formulate new ideas as shown earlier in this article. It is anticipated that gifted groups will be able to work toward advanced placement in the senior high school, as in this group are future professional mathematicians, scientists, engineers, and physicists.

The *Symbol of Multiplication* (×), St. Andrew's cross, was introduced by William Oughtred, an eminent English mathematician and divine, born at Eton in 1573. The work in which this symbol first appeared was entitled *Clavis Mathematicae*, "Key of Mathematics," and published in 1631. Oughtred was a fine thinker, and was honored by the title "prince of mathematicians."

The *Symbol of Division* (÷), was invented by Dr. John Pell, Professor of Philosophy and Mathematics at Dreda. He was born at Southwick in Sussex, 1610, and died in 1685. This symbol was used by some old English writers to denote the ratio or relation of quantities. I have also noticed it used thus in some old German mathematical works. Dr. Pell was highly regarded as a mathematician. It was to him that Newton first explained his invention of fluxions.—EDWARD BROOKS, *The Philosophy of Arithmetic* (Lancaster: Normal Publishing Co., 1880), p. 110.

A new approach to an old problem

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Action is seldom taken until "a blow strikes home!"

An analysis of test scores indicated a downward trend in arithmetic achievement—a trend that began in the fourth grade. However, a general survey of the arithmetic situation led to the following conclusions:

I. Low arithmetic achievement in middle and upper grades has become a problem of state-wide concern.

II. California educators have begun to examine curriculum practices in other states and in other countries, and many districts are formulating plans to accelerate the teaching of arithmetic.

III. There is a growing feeling that California schools have been "too easy," particularly in the area of arithmetic.

IV. Large classes and the common practice of heterogeneous grouping have made it increasingly difficult for teachers to adequately meet individual needs.

V. An alarming number of middle and upper elementary teachers assign the same lesson in the state textbook to the entire class, thus frustrating slow learners and retarding those who could be advancing rapidly.

It took these disturbing results of a district-wide achievement testing program to establish the fact that Lancaster School District was seriously low in arithmetic achievement. Some kind of action was inevitable and urgent, because this blow did "strike home." The staff accepted the challenge to develop a plan which would accelerate the arithmetic program and place Lancaster School District among those which are trying a new approach to the currently discussed problem in arithmetic curriculum.

Pilot studies

The writing of an arithmetic guide in which all new processes were introduced from six months to a year earlier was undertaken. Plans for two pilot studies were formulated to determine the grouping procedure best suited to the needs of the district.

Pilot Study #1 followed the plan of homogeneous grouping within grades four, five, and six, with emphasis on horizontal enrichment. Although there were many points in its favor, teachers still had a wide range of ability within groups and found the meeting of individual needs very difficult. Rapid learners were confined to a grade level. Since one of the main objectives of the pilot studies was to find a plan which would accelerate capable students by introducing arithmetical processes at an earlier age, it seemed illogical to limit such students to a grade level beyond which they would not be allowed to progress.

Vertical acceleration was practiced in Pilot Study #2. This plan was based on the philosophy that the school should accept the student where he is in academic achievement and make it possible for him to advance as far as he is capable of going. All fourth-, fifth-, and sixth-grade students were tested and placed in ability groups,

irrespective of grade level, for a period of special arithmetic instruction at the same time each day.

The purpose of this procedure was to determine the most suitable plan of grouping students for special arithmetic instruction in the intermediate grades in order to: (*a*) provide opportunities for acceleration of rapid learners, (*b*) better teach average students, and (*c*) reteach students who failed to grasp certain arithmetic processes when they were presented in earlier grades.

Procedure

Much of the success of this plan of grouping for arithmetic instruction is determined by the manner in which it is introduced and co-ordinated. Following are the main points of procedure as the plan was undertaken at Sierra School in Lancaster:

I. The principal explained the program to his fourth-, fifth-, and sixth-grade teachers. Several meetings were necessary to formulate plans for testing and placing of students.

II. The P.T.A. president was asked to assist by calling a meeting of the parents of those children who were to be involved in the pilot study. The program was described to the parents and their questions were answered by the principal and his teachers.

III. The California Achievement Test was administered to all fourth-, fifth-, and sixth-grade students. Test results, previous records, and teacher judgment were used as criteria in assigning students to initial ability groups.

IV. Before assignments were mentioned to the students, the principal visited each regular class and explained, to the satisfaction of the students, how the program would operate and what was meant by being assigned to a group "according to one's needs."

V. Each teacher was assigned an ability range and was held responsible for teaching all processes in the corresponding section of the new Lancaster School District Arithmetic Guide.

VI. Parents and students were told that initial pupil assignments and any subsequent placement of students would be entirely flexible and that students could progress from one group to another as soon as they were capable of doing the work.

The following points of classroom procedure evolved from group planning:

I. Each student was required to keep a graphic record of his own progress and a folder of his work. This folder was kept in the teacher's file and was given to the pupil only when papers were to be added or when test scores were to be recorded.

II. If and when a student had qualified for a higher level, the two teachers met to discuss his work before he was told. This prevented student disappointment, if the transfer was to be delayed.

III. When a student transferred, he took his record of progress and his folder of classwork to the next room where it was placed in the teacher's file.

IV. Remedial groups spent much time learning fundamental skills, while accelerated groups were engaged in arithmetic projects involving advanced mathematics. However, all groups were required to spend at least one day a week on arithmetic drill in some form.

V. Tests were given on the same day in all groups and resulting scores were recorded by the pupil on his individual progress chart.

Parents did raise questions about the study as it developed. The following are illustrations of these questions and the answers given:

1. How will you grade my child in arithmetic?

Answer: He will be graded according to his effort and progress within his group. If he is doing well, he will receive a good grade, but his teacher will indicate whether he is working at grade level, below grade level, or above grade level.

2. Will this program of acceleration be continued in the upper grades?
Answer: Yes, plans are being formulated to take these students right on into an accelerated program in the seventh and eighth grades.
3. My child is in the fourth grade and you have placed him in sixth-grade arithmetic work. How will he get fifth-grade arithmetic? Won't he miss out on many vital processes?
Answer: Each group will go back regularly to review fundamental processes. Your child has shown that he has the ability to learn these processes quickly. He will be able to progress far beyond his grade level, but constant review will insure against his missing out on any important process.
4. What will happen to my child if we move to a district in which there is no accelerated program?
Answer: It is the policy of any good school system to accept a child where he is in scholastic achievement and to make it possible for him to advance as far as he is capable of going. Your child should be better able to adjust to a new situation after having had the advantage of participating in this program.
5. Johnny has always received A's in arithmetic! Now, he is in this fast group and he came home with a C on his report card. We don't understand this sort of thing. We want our boy to make good grades.
Answer: Mrs. Smith, your boy is working with an entire group of "good students." Which do you think is worth more to him—a C among high students or an A in the group in which he doesn't have to exert any extra effort to get the best grade in the class?

Evaluation by standardized tests

Test results summarized in the following statements were taken from the records of Sierra School:

I. Retesting with the California Achievement Test Arithmetic Section showed an over-all average gain of three days achievement for every day of instruction for the period of the pilot study.

II. The S.R.A. Arithmetic Achievement Test was given to all fourth-, fifth-, and sixth-grade students at the beginning of the current school year. Retesting after a period of 29 days of instruction produced results which further attest to the success of grouping for arithmetic instruction. Average gains by grade are:

Fourth grade	8 months
Fifth grade	5 months
Sixth grade	$3\frac{1}{2}$ months

Individual high gains made in the 29-day period are as follows:

Fourth grade	3.1 years
Fifth grade	3.5 years
Sixth grade	1.9 years

Sixty-four pupils made more than a year's gain in the 29-day period as compared to thirty-seven who did not show a significant gain.

III. Although fourth-grade students have needed a comparatively longer period to become adjusted to the program and to changing rooms and teachers, test results show a greater gain in achievement on the average than for fifth or sixth graders.

Evaluation by students

Comments made by students were:

"I like my arithmetic class because we get to work on projects like going outside to measure play areas after we finished studying about geometric figures. Next, we are going to plan the building of a home." (Accelerated Group)

"No one makes fun of me now when I drill on my multiplication tables, because

everyone in the class is in the 'same boat.' I used to pretend that I knew them when I didn't because I was afraid the other kids would call me stupid." (Remedial Group)

"We all work together in my arithmetic class. I used to have a hard time finishing my written work because another group would be too noisy, but now the room is quiet when we have study time." (Low Average Group)

"At first, I was scared that I wouldn't like another teacher for arithmetic, but now I think it's fun!" (Fourth grader)

"Having more than one teacher is like Junior High School." (Sixth grader)

"Mrs. Smith makes arithmetic seem different—more fun or something. Guess she just likes to teach arithmetic!" (Fifth grader)

"We don't have to wait for slow kids." (Remark frequently heard in accelerated group)

Evaluation by teachers

Comments made by teachers were:

"For the first time in my teaching career I feel as if I'm doing a good job of teaching arithmetic, because I don't have to cope with so many levels of ability. Test scores show a range of arithmetic ability in my regular class from 1.0 to 6.5 (or a range of $5\frac{1}{2}$ grades). My assigned group for the arithmetic program has an arithmetic ability range from 3.5 to 4.0 (or a range of $\frac{1}{2}$ grade). What a difference it makes!"

"The change in classes during the day is both challenging and refreshing to teachers and students."

"Knowing that I have to meet a different group of children for the arithmetic period seems to force me to spend more time in preparation, and I'm certain to become a better teacher as a result."

"At first, I was concerned about my fourth graders and whether their immaturity would make it too difficult for them to participate in this program. Just the opposite has proven to be true! They look forward to the arithmetic period and to the chance to go to another room. It appeals to their ego and seems to make them feel very grown-up."

Evaluation by the principal

The principal made the following statement:

I. Many administrative problems arose as this program actually began to function, but the teachers and I worked as a team to develop the most workable procedures for our school.

II. Retarded students seem to react to the program in one of two ways:

A. Those who are capable, but lazy, seem rather quickly to "find" themselves. There is a sudden show of determination to achieve —motivated perhaps by the desire to be placed back in their peer group.

B. Those of *low ability* seem happy to be in a group in which it is possible to gain recognition on their level. Those who have been discipline problems in their regular classrooms have "settled down" in this program.

III. Rapid learners are no longer free to "loaf" as they are sometimes permitted to do in a regular classroom when they have quickly finished the assignment and have simply wasted the rest of the period, helped slow students, or performed some janitorial task for the teacher who couldn't think of any other way of keeping them busy. In this program, they are competing against a group of students of like ability and have to "produce" or get left behind. Needless to say, a few who have been used to staying ahead of the class without exerting any great amount of effort have found this program a bit frustrating, but they have done very well after once realizing that they will have to work harder.

IV. I don't believe this program puts undue pressure on students or causes immature students to feel insecure. I have made it a point to carefully observe the few students whose parents have complained and, in each case, I have reached the conclusion that it was the parent who was unhappy instead of the child—usually because the child had not been placed in a "more advanced group."

V. In this program, using this system of grouping for arithmetic instruction, no parent can justly complain that his child is being neglected. Every level of ability receives special attention, and the rapid learner can advance as far as he is capable of going.

VI. A comparison between ranges of ability in regular classes and those of the arithmetic classes will serve to explain why teachers can better meet individual needs in the arithmetic program.

Teacher	Regular class	Arithmetic class
#1	3.1–8.4 (5.3)	.7–3.0 (2.3)
#2	.7–4.8 (4.1)	3.1–3.5 (.4)
#3	1.0–6.5 (5.5)	3.5–4.0 (.5)
#4	.7–5.4 (4.7)	4.0–4.8 (.8)
#5	2.4–6.8 (4.4)	5.0–5.5 (.5)
#6	3.9–9.5 (5.6)	5.6–6.4 (.8)
#7	1.9–7.6 (5.7)	6.4–9.5 (3.1)

Evaluation by the curriculum staff

Many benefits to the entire district have been derived from the functioning of the Lancaster School District Accelerated Arithmetic Program:

I. Good public relations experiences have evolved from parent group meetings.

II. Unity in the district has been further established through co-operation among district personnel in the effort to achieve a common goal—that of upgrading the district arithmetic program.

III. Teachers have become interested in learning better methods of teaching arithmetic.

IV. Principal-teacher planning has helped to develop a higher level of teacher morale. Over-all achievement scores have indicated a carry-over of improved teaching procedures to other subject areas.

V. The emphasis placed on adequate meeting of individual needs in this program has influenced other grade levels to stress its importance.

Conclusion

All indications are that Lancaster School District has developed a plan of grouping for arithmetic instruction in the intermediate grades which is successful for this area. The program is in its second year of operation at Sierra School, and it was adopted on a district-wide basis at the beginning of the current school year.

We are aware of the fact that programs which have succeeded as pilot programs have sometimes failed when made a part of the regular district curriculum, due to a lessening of interest and enthusiasm. Plans are being made to keep arithmetic "in focus" over a longer period of time. The continued success of the program will depend largely on:

I. Adequate co-ordination and supervision by administrative personnel

II. Proper motivation of teachers and pupils

III. Continued interest and co-operation of parents

IV. Good teaching procedures and techniques

V. Effective uses of teaching aids and tools

However, at the present time, Lancaster's new approach to an old problem—that of how to introduce arithmetic earlier and teach it better—seems to be producing unusually gratifying results. It may not be the best plan, but it is a plan that works!

Arithmetic instruction changes pupils' attitudes toward arithmetic

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The fact that pupils in all arithmetic classes differ in such ways as understanding of number operations, rate of learning, background of experience, and interest in the subject indicates that arithmetic instruction should be adjusted to provide for the apparent variations in pupils. Authors of texts concerned with the teaching of arithmetic recognize that these pupil differences exist and suggest methods of teaching and classroom organization to provide for them. Classroom teachers are also aware of the extent of these differences and are concerned with methods of instruction which will meet the needs of all the pupils in their classrooms. The adjustment of arithmetic instruction to variations in pupils has become a major aspect of arithmetic instruction. Most of the proposals and methods for adjusting arithmetic instruction to pupil differences in arithmetical abilities and understandings are concerned with some organizational procedure and involve some type of ability grouping or some type of individualized program.

Significance of pupil attitudes in arithmetic instruction

The importance of adjusting arithmetic instruction to pupil differences should not be discussed without also considering the importance of developing and maintaining favorable pupil attitudes toward arithmetic. Experiences in arithmetic classes play a major role in the development of attitudes toward arithmetic. Such attitudes are developed early in the child's school experiences and are significantly related to achievement in arithmetic.

Methods of adjusting arithmetic instruction to individual differences by individualizing instruction or by grouping have been severely criticized as adversely affecting the attitudes of the pupils and the parents involved. Critics of grouping methods claim that pupils in the lower grouping will suffer a loss of self-respect and that parents will denounce such classroom organizations as undemocratic procedures. No parent wants his child to be classified as a slow learner. Critics further claim that such feelings on the part of parents and children will have an unfavorable effect upon the child's attitudes toward arithmetic and the study of arithmetic. Proponents of organizational methods for adjusting arithmetic instruction to pupil differences suggest that a child's success at his own level of ability in arithmetic will have a distinct, favorable effect upon his attitudes toward that subject, and that success for each child can be better achieved through certain organizational procedures.

Measuring the effects of grouping on pupil attitudes

Investigations in this area have dealt with the effect of certain practices on

pupil attitudes toward the subject, and on schoolwork, and have generally been associated with studies concerned with gains in arithmetic achievement. A review of the findings of some of these studies should give some indication as to whether the claims of the critics or of the proponents of such methods can be verified. The findings of such studies are based largely upon information gathered from questionnaires completed by pupils, parents, or teachers.

In a study which investigated the value of individualizing instruction for remedial arithmetic with ninth-grade pupils, Bernstein[1] found that many pupils expressed a greater liking for mathematics after they had participated in the individualized program. When Justman[2] examined the attitudes of pupils enrolled in special progress courses and pupils enrolled in normal progress courses, he found that there were no significant differences in attitudes and that the special progress courses had no ill effects upon the attitudes of the pupils. Kvaraceus and Wiles[3] looked at this problem in a somewhat different fashion. They ascertained that when pupils were classified into groups according to their achievement and apparent abilities, disciplinary problems were reduced and pupils' attitudes toward the study of the subject area seemed to have been improved. Holmes and Harvey[4] found that the method of grouping, whether flexible or permanent, appeared to have no differential effect on the attitudes of the children toward arithmetic. However, in both types of grouping, the attitudes of pupils toward arithmetic were less favorable at the end of the year than at the beginning.

In a more comprehensive study of attitudes in relation to intra-class grouping for arithmetic instruction, Spence[5] considered the attitudes of parents, pupils, and teachers as revealed by questionnaires. Parents believed that subgrouping made the study of arithmetic easier, more enjoyable, and more meaningful for their children, reduced the amount of home assignments, was liked by their children, and should be continued in the future. In general, pupils in subgrouped classes believed that arithmetic was made easier and more enjoyable, that they received more help from the teacher under such conditions, that they accomplished more with fewer home assignments, and that the grouping was desirable and should be continued. The teachers involved believed that subgrouping was successful for all groups, that it required more planning and more materials, that better provision was made for individual differences, that there were no antisocial feelings, that children liked this type of classroom organization, and that subgrouping for instruction should be continued.

A recent investigation

In a study conducted by this writer,[6] the changes in attitudes toward arithmetic of pupils taught in grouped arithmetic classes were compared with the changes in attitudes toward arithmetic of pupils taught in a traditional nongrouped situation. The experimental group consisted of two fourth-grade classes which were grouped and regrouped when new topics of arithmetic were to be studied. The two contrast classes were taught in nongrouped situations. Pre- and post-inventories of attitudes toward arithmetic were administered to each group. Changes in attitudes

[1] Allen L. Bernstein, "A Study of Remedial Arithmetic Conducted with Ninth Grade Students" (Unpublished Doctoral Dissertation, Wayne University, 1955).

[2] Joseph Justman, "A Comparison of the Functioning of Intellectually Gifted Children Enrolled in Special Progress and Normal Progress Classes in Junior High School" (Unpublished Doctoral Dissertation, Columbia University, 1953).

[3] William C. Kvaraceus and Marion E. Wiles, "An Experiment in Grouping for Effective Learning," *Elementary School Journal*, XXXIX (December, 1938), 264–68.

[4] Darrell Holmes and Lois Harvey, "An Evaluation of Two Methods of Grouping," *Educational Research Bulletin*, XXXV (November, 1956), 213–222.

[5] Eugene S. Spence, "Intra-Class Grouping of Pupils for Instruction in Arithmetic in the Intermediate Grades of the Elementary School" (Unpublished Doctoral Dissertation, University of Pittsburgh, 1958).

[6] Harold H. Lerch, "A Study Concerning the Adjustment of Arithmetic Instruction to Certain Individual Differences" (Unpublished Doctoral Dissertation, University of Illinois, 1960).

were determined and a comparison of the changes in attitudes toward arithmetic between the two groups was made. As measured by the instruments used in this study, the changes in attitudes toward arithmetic of the experimental classes were not significantly different from the changes in attitudes toward arithmetic of the contrast classes. In both groups, more than one-half of the pupils indicated changes to more favorable attitudes toward arithmetic. The experimental grouping procedure had no more adverse effect upon the pupils' attitudes toward arithmetic than did the more traditional programs of the contrast group.

Reports by Ivie, Fowler, and Graham[7] and by Ivie, Gunn, and Holladay[8] do not completely support these findings. They indicate that some pupils are dissatisfied with working in small groups and would rather work with the class as a total group.

Importance of teacher attitudes in relation to building favorable pupil attitudes

The few studies reviewed here certainly do not verify the criticism that classroom organizational procedures of grouping or individualizing instruction for arithmetic have an adverse or unfavorable effect on pupils' attitudes toward arithmetic. Neither do the findings of these studies verify the supposition that organizational procedures of grouping or individualizing instruction for arithmetic will develop more favorable attitudes on the part of pupils. It should be recognized that organizational procedures, as such, do not solve the problem of adjusting arithmetic instruction to pupil differences, or of developing and maintaining favorable attitudes toward the subject. The child's successes in arithmetic and his attitudes toward arithmetic are more basically dependent upon his teachers' attitudes and the methods they employ than they are upon classroom organization.

The importance of developing and maintaining desirable attitudes toward arithmetic suggests that teachers at all grade levels should be aware of their pupils' attitudes toward the subject and should strive to use teaching methods that will help develop favorable attitudes toward arithmetic. It would seem that if desirable attitudes toward arithmetic are to be developed and if undesirable attitudes are to be changed, arithmetic pupils should be assured a certain measure of success. Thus, classroom organizational procedures and teaching methods should be ones which assure each pupil a measure of success at his own level of ability and understanding, and which at the same time encourage the development and maintenance of favorable attitudes toward arithmetic.

[7] Claude Ivie, Eugenia Fowler, and Virginia Graham, "Grouping in the Normal Mathematics Class," *The Mathematics Teacher*, LI (October, 1958), 450–52.

[8] Claude Ivie, Lilybel Gunn, and Ivon Holladay, "Grouping in Arithmetic in the Normal Classroom," THE ARITHMETIC TEACHER, IV (November, 1957), 219–21.

Science and mathematics teaching in elementary and secondary schools was recently criticized by Dr. Edward Teller, the nuclear physicist who played an important role in developing the hydrogen bomb. He blamed unimaginative instruction for a large part of the "loss" of outstanding students to other less demanding fields. Science and mathematics courses are too frequently taught as dull exercises in fundamentals rather than as intellectual adventures and so "fall short of the spirit of the subject," he said. Dr. Teller suggested that higher mathematics and science be taught to pupils at the age of 10 or 11. He also urged that "honors" be granted to teachers whose teaching produced superior students.

—From *Education Summary*, November 12, 1960, p. 1.

Grouping by arithmetic ability—an experiment in the teaching of arithmetic

GRANT C. PINNEY *Public Schools, China Lake, California*
Mr. Pinney is assistant superintendent of China Lake schools.

That children in the regular elementary classroom at every grade level vary widely in their interest and achievement in arithmetic is a well-known and accepted fact. With this fact in mind, the staff at Vieweg Elementary School felt that most of the difficulty in teaching arithmetic was due to the great span of ability found in a heterogeneous classroom of fifth- or sixth-grade children.[1] In a sixth-grade class the grade placement ability on achievement tests ranged from 2.0 grade level to 10.0 grade level—eight full years. The problem, as we saw it, was to reduce the range of arithmetic achievement ability in each classroom.

In the fall of 1958 the Vieweg Elementary School fifth- and sixth-grade teachers launched a program in arithmetic that we felt would improve achievement level of fifth- and sixth-grade arithmetic students. We found that we could cut the arithmetic ability range in half by grouping the students according to arithmetic ability. We put the top thirty students in one class and the lower half of the children in another class. The only criterion used for grouping was arithmetic ability according to scores on the S.R.A. achievement tests for grades four to six. This gave us a class load of approximately thirty children per class. Children were transferred from class to class on teacher recommendation two times during the year. When new students were enrolled they were placed in the class according to reports from previous schools regarding arithmetic grades and ability scores. A test was given to students of doubtful ability.

To carry on this program we knew that we must allow the slow learner more time on successive topics and thereby postpone the presentation of some topics until later in the year or even in the following year. We knew also that we must give shorter assignments to slow learners. Assigning special homework gave the slow learners additional practice. The faster workers must be given additional arithmetic enrichment activities and must be allowed to move on to new topics at a faster rate of speed.

Charts 1, 2, 3, and 4 show the ability range of the students in both the regular class and the class formed by grouping according to arithmetic ability and the achievement level of these students at the beginning of the program and at the end of the program. For comparison we used the sixth-grade classes only, as we had no previous scores on the fifth-grade students.

The results as shown by the foregoing charts were above our expectations. In one year the students had grown more than two years in arithmetic achievement. The growth in arithmetic reasoning was from 4.7 grade level to 6.8 grade level. In arithmetic concepts the growth was from

[1] The Vieweg Elementary School is a small school. It consists of two classrooms of each grade from kindergarten through sixth grade. At the time of this experiment fifty-eight students were enrolled in the sixth grade and sixty students were enrolled in the fifth grade.

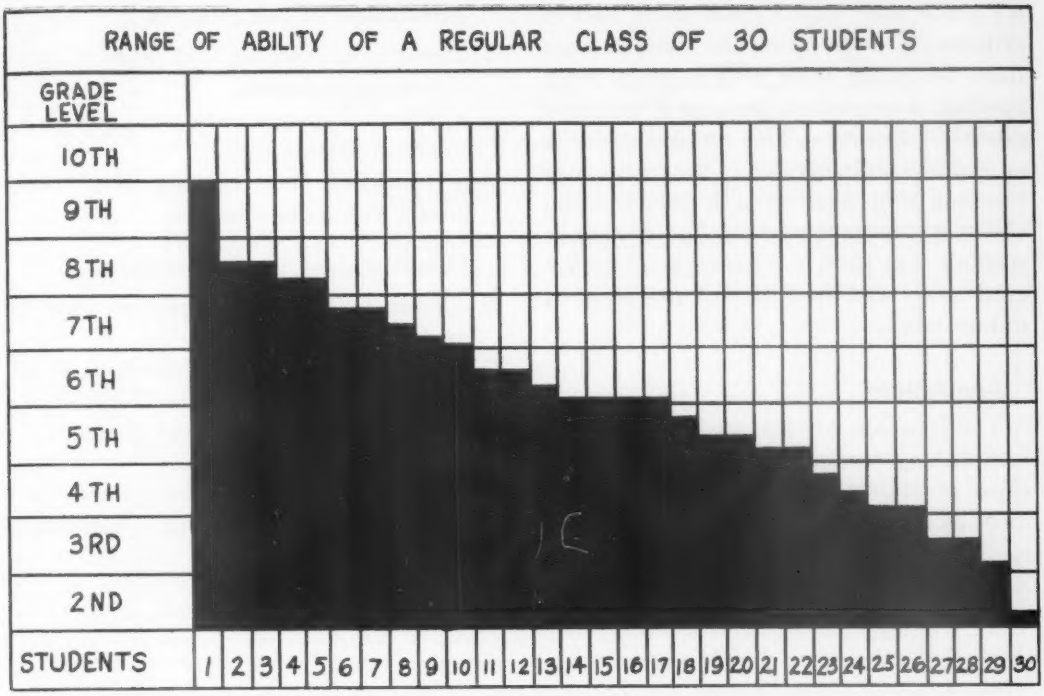


Chart 1 shows the range of arithmetic ability of one of the sixth-grade classes before the children were separated according to arithmetic ability. You will notice that the class charted above had a range of ability of 8 years with only 30 students measured.

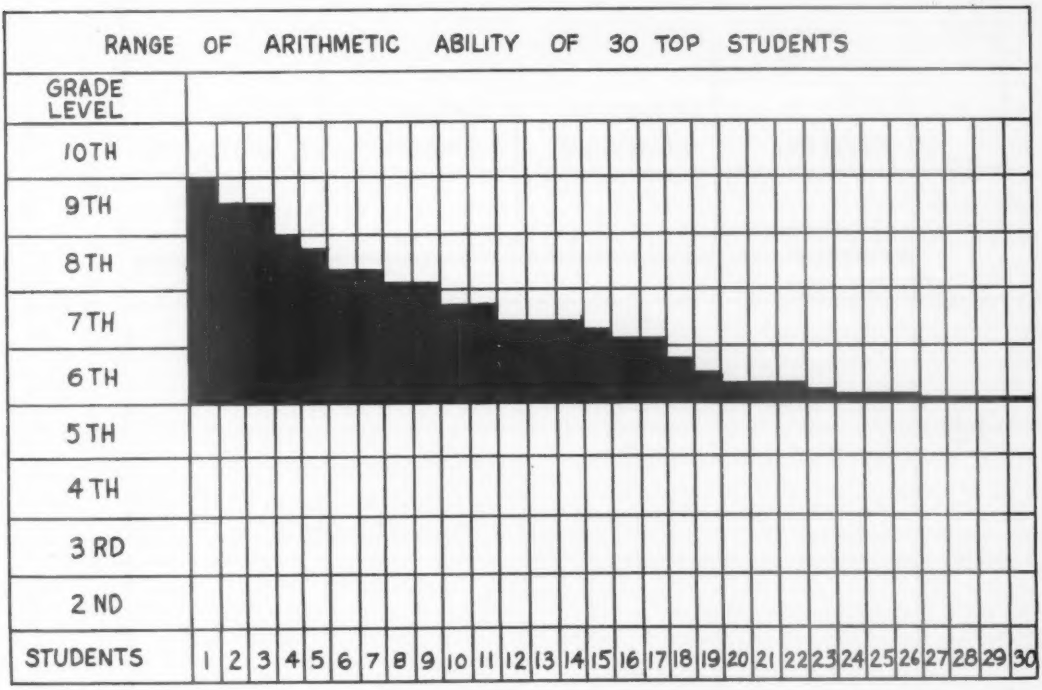


Chart 2 shows the range of arithmetic ability of the top class after the separation has been made according to arithmetic ability. You will notice here that the range of ability for this group of students is just four years.

5.1 grade level to 7.3 grade level, and in arithmetic computation the children grew from 5.0 grade level to 7.3 grade level. Spelling is shown here because of the great growth in this area. This was a surprise to us and was probably due to the methods of teaching that we were able to use in an ability-grouping situation. The growth in spelling was from 4.6 grade level to 7.4 grade level and spelling was not stressed in any way.

Conclusions

The program was a success as far as accomplishing the goal we had set. At the close of the school year an evaluation meeting was held and the following conclusions were made:

Advantages of the program

1. Great improvement was made in academic achievement.
2. Children were more at ease in the classroom situation.
3. Preparation and teaching were made easier and more effective.

Disadvantages of the program

1. Eliminates leadership from one class to some extent.
2. Low group must be a formal class all day to cover all subjects. This allows little time for committee or group work, class meetings, etc.
3. High class received honors during the year and low class did not. Some competition was evident.

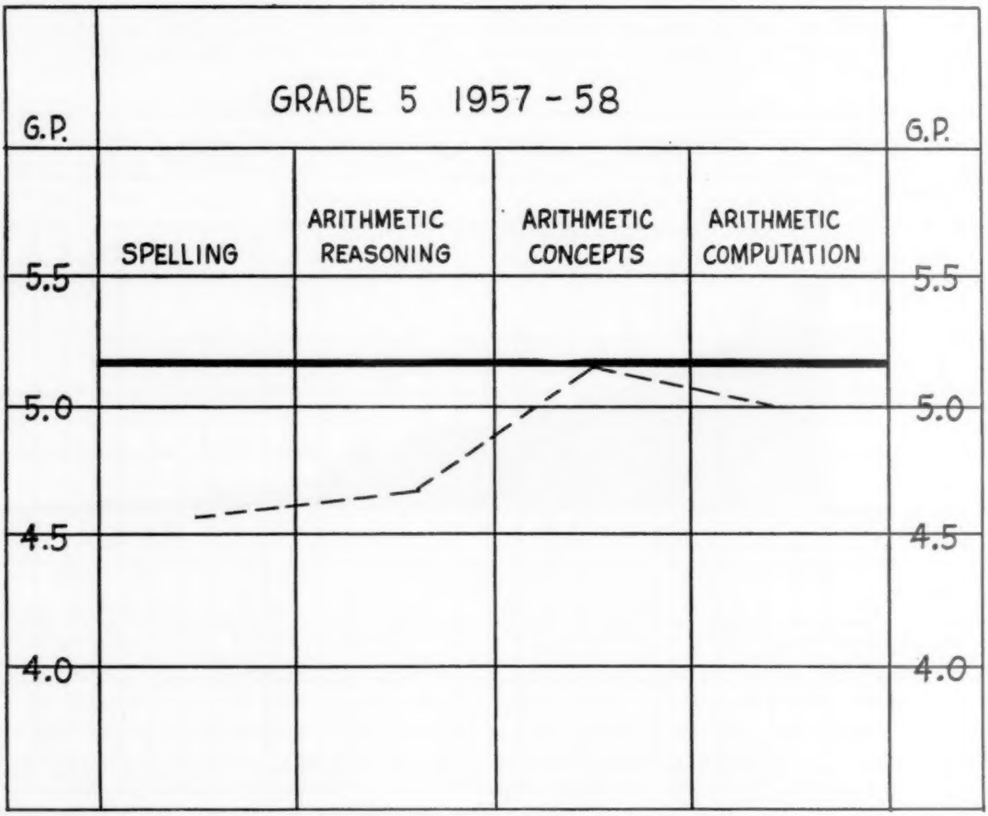


Chart 3 shows the achievement level of the 58 students before beginning the program. The heavy line shows the national norm and the dotted line shows the achievement level of this group of students.

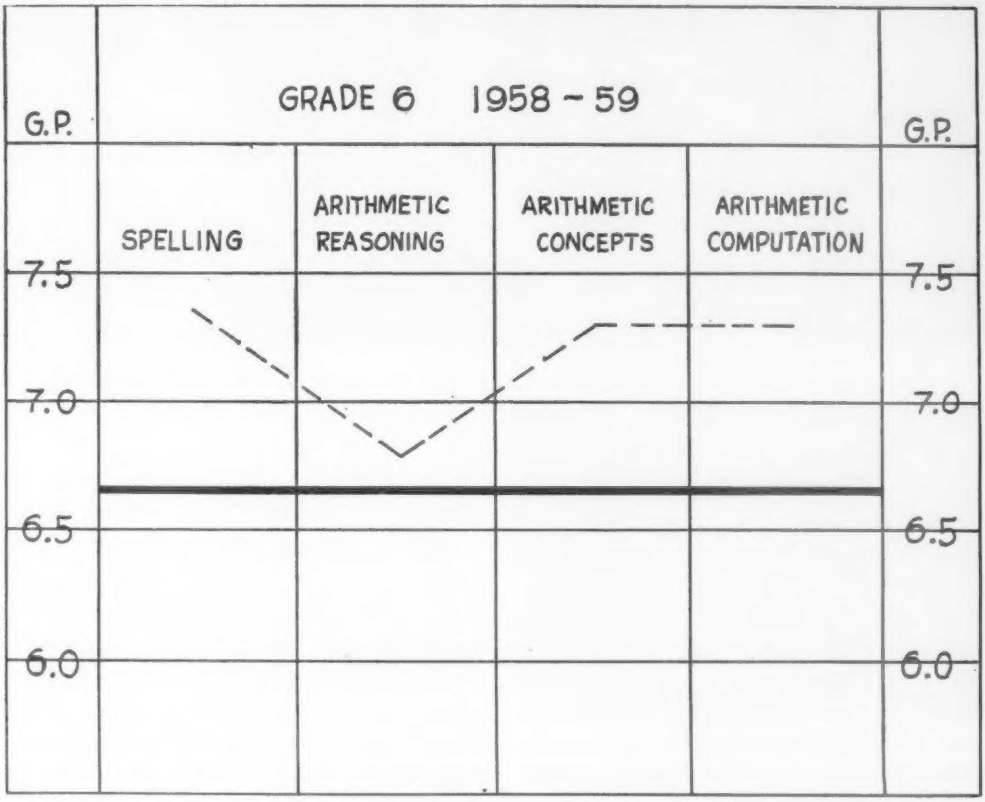


Chart 4 shows the achievement level of the 58 students after one year in the program. The heavy line shows the national norm and the dotted line shows the achievement level of this group of students.

4. Frustration to the teacher of the low group. Takes time for teacher to accept the slower achievement of class and the lower achievement that can be expected of the class.

Suggested recommendations for another year

1. Repeat the program if teacher of low group is willing to accept and work at this level again.
2. If teacher is not willing to accept this low group with enthusiasm, the children should be grouped the same way for just one hour a day during the arithmetic period.
3. One hour of arithmetic should be taught in the low group and not less than forty-five minutes a day in the high group.
4. Low group could use more audio-visual material during the year, such as records, charts, flash cards, etc.

To the editor

There appear to be several errors on page 396 under b in the December issue of THE ARITHMETIC TEACHER.

GEORGE GREENOUGH
Escondido, California

Yes, there is an error. The statement should read, "Team A won one out of *four* games before Christmas and four out of seven games after Christmas." Thanks for calling this to our attention.

THE EDITOR

Developing concepts of time and temperature

The use of daily ceremonies in the primary grades

IDA MAE HEARD *Acadia Parish Schools, Crowley, Louisiana*
Dr. Heard is mathematics supervisor for the Acadia Parish Schools.

A well-balanced program in mathematics for the primary grades includes many opportunities for children to acquire concepts of measurement. Some teachers achieve this balance by reserving one day of each week for developing measurement ideas. Other teachers schedule some measurement experiences and capitalize on any unexpected situation that arises. Quite a number of teachers depend on the repetition of some worthwhile daily experience to give meaning to some phase of measurement, such as time or temperature.

Any good teacher uses a variety of ways to provide firsthand experiences that tie certain measurement notions to the child's daily environment. The following photographs made by the author show some of the recurring experiences used by the primary teachers of the Acadia Parish Schools to teach time and temperature.

Learning to tell time

The telling of time on the hour can be greatly simplified by using the hour hand *only* at first. Demonstration clocks with instructions for the teacher and pupils can be obtained free from:

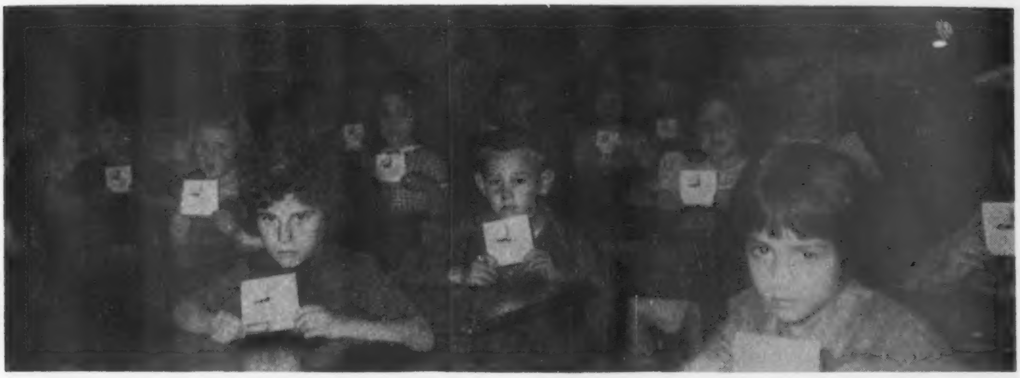
The Watchmakers of Switzerland
Information Center, Inc.
730 Fifth Avenue
New York 19, New York

The teacher uses a large model and the pupils use smaller clocks. After the hour hand is attached, the children learn to show the time *on* the hour, a little *before* the hour, and a little *past* the hour. Finally, they learn to show the position of the hour hand at half past the hour, at a quarter past the hour, or at a quarter to the hour.

The pupil moves the hour hand to show its position when school is out.

The first graders use both hands on their clocks to tell the time that school starts.

"We go home at half past three."



"We start school at 9:00 o'clock."

"We start for school at 8:00 o'clock."

A second-grade teacher made an experience chart by mounting the small clocks on a sheet of poster paper.

Keeping a day-by-day calendar

There are many things that children learn by keeping a progressive calendar. They become aware of the proper sequence of the numbers from 1 to 31. The youngsters learn the ordinals from "first" to "thirty-first." They learn that most months have 31 days, that some months have 30 days, and that February is the shortest month. The pupils also learn the order of the days of the week and the number of days in a week, and finally they learn the approximate number of weeks in a month.

The pictures on page 126 show various ways of having the children keep a day-by-day calendar.

As each child took his turn in crossing off a day, he told the day of the week, the month, and the year. Most teachers used a calendar that was purchased from a commercial source.

Learning to read a thermometer

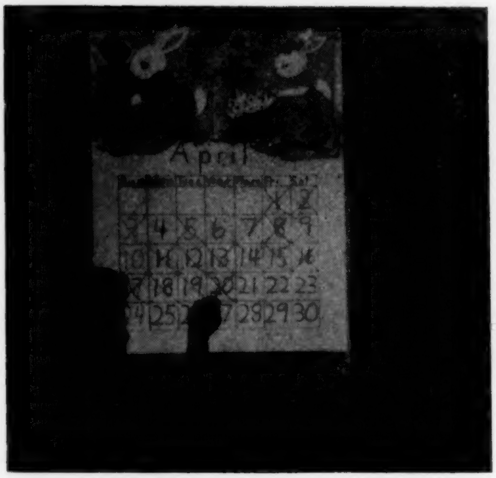
All the teachers had an educational thermometer, and the children learned to read the scale in units of 2 degrees.

In the second-grade classes, the children had read a news story in their *Weekly Reader* on "Summer In Antarctica" (Edition Two, January 18–22, 1960). From this issue they also found that water freezes at 32 degrees. They located this point on their thermometer.

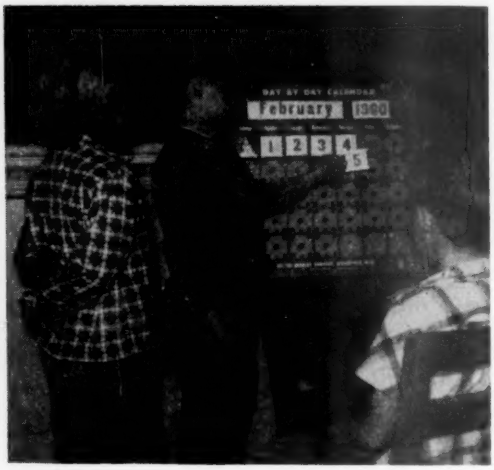
One second-grade class kept a chart of the temperature readings at 9 A.M., at noon, and at 3 P.M. They compared the temperatures from one reading to the next and from day to day at the same hour. In general, they found the noon temperature higher than the one at 9 o'clock. There were a few times during the fall and spring months when the outdoor and indoor temperatures were found to be the same.

Making a broken-line graph

During the month of February, a group of third graders recorded the indoor and outdoor temperature readings of their classroom at 9 o'clock each morning. With the help of their teacher, the children



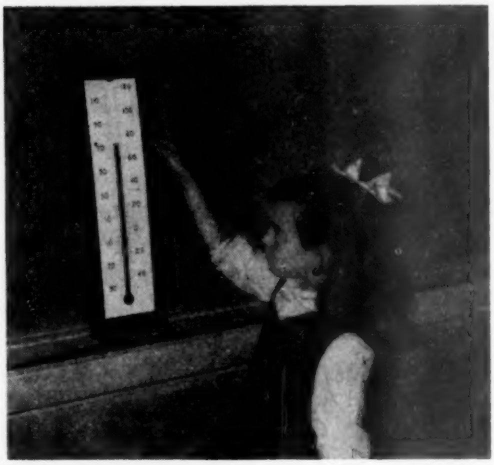
Crossing off the days.



"Today is February 5, 1960."

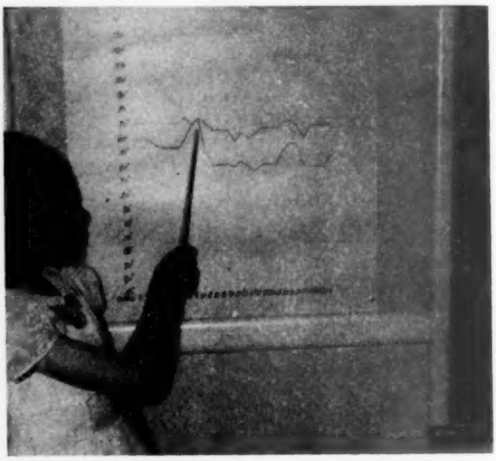
made the graph pictured which shows the daily changes in temperature. A red line was used to represent the inside temperature, and a blue line represented the outdoor readings.

The daily ceremonies described here are one means of gradually unfolding ideas about time and temperature. These concepts are not only useful to the children at the time they learn them, but they form a stockpile of information which can be drawn upon in the solution of oral or written problems.



"The temperature of our classroom should be kept at about 72 degrees."

Learning to compare the readings on the indoor-outdoor scales is shown here.



"On February 9, the indoor and outdoor temperatures were the same."

Ordered pairs, patterns, and graphs in fourth grade

LUCY E. DRISCOLL *Cook County Public Schools, Chicago, Illinois*
Miss Driscoll is assistant superintendent of schools in Cook County, Chicago, Illinois.

How many cuts will you make to cut a board into four parts?" "If Tommy is four years old when his father is twenty-four, his father is how many times as old as Tommy?" Although in itself this presented no apparent difficulty, someone then volunteered the statement, "Then when Tommy is six years old, his father will have to be thirty-six." At this point there was some shaking of heads, much discussion, and a number of seemingly perplexed children.

These two problems and the discussion which followed in a fourth-grade class were interesting problems dealing with "ordered pairs" and the way these pairs formed patterns. They were the motives that suggested the following end experiences to the writer. Might it be possible to help these children to picture these patterns as mathematical graphs? Briefly the "experiment" was as follows.

Exploration

The two problems stated above were discussed and continually rephrased until the pupils themselves clearly saw patterns evolving. The suggestion was made that for problems in which pairs of numbers are involved and in which those pairs "move" together in some way, it is always possible to make a "picture" in the form of a graph. Since the class had had experience with temperature graphs, this was used as our starting point. Along the horizontal line (the word "axis" was not used) we wrote the dates of the month and along the vertical line the temperatures. (See Figure 1.) We then decided upon the unit to be used, and proceeded to write the 0 at the intersection of the two axes. At equal intervals 5, 10, 15, 20, etc. were written. We then "plotted" the temperatures we had assigned to the various dates.

The result of our finished graph was, of course, a broken line graph, but this had given us experience in plotting points to represent the pairs of numbers. We had even found that we could "interpolate" as needed (and they thoroughly enjoyed these big, new words, too).

Next we talked of the number of cuts necessary to cut the board into several parts, one of the two problems that had prompted this experience. We decided to write along one line the number of cuts, and along the other the "partners" of these numbers—the number of pieces of board which would result. (See Figure 2.)

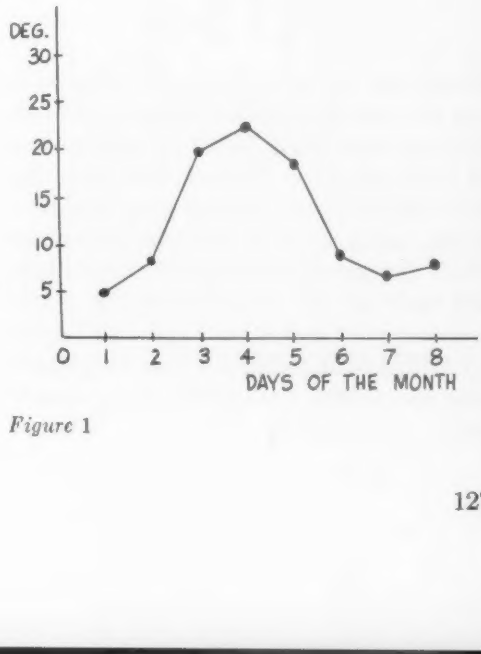


Figure 1

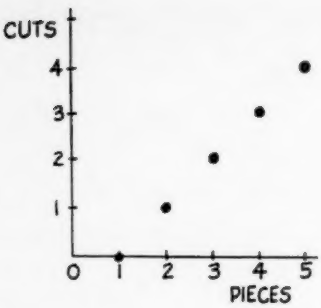


Figure 2

We now plotted these pairs of numbers. We didn't draw a line, for this would suggest we had parts of cuts as well as parts of pieces. We later talked about it: that we had plotted several points representing pairs of numbers such as one cut, two pieces; two cuts, three pieces; three cuts, four pieces; these points helped us name other pairs of points, etc.

Next we talked about the second problem, which dealt with the ages of father and son. One boy spoke up and said, "Yeah, I never did quite understand that one." We decided to have Father be 25 when Tommy was one year old, and we organized in two columns their ages over the next several years—then "skipped" to the age both would be when Tommy would be 5, 10, 15, and 20 years old.

TOMMY	FATHER
1	25
2	26
3	27
4	28
5	29
10	34
15	39
20	44

Examining our two columns of figures, it was evident that Father would always be 24 years older than Tommy, since he was 24 years old when Tommy was born. We now plotted these pairs of numbers on a graph, using units of one year along the x-axis (horizontal line) for Tommy's age, and units of five years along the y-axis (vertical axis) for Father's age. (See Figure 3.) After plotting several points we realized that the points were falling in a certain way.

TOMMY	FATHER	TIMES AS OLD
1	25	25
2	26	13
3	27	9
4	28	7
⋮	⋮	⋮
6	30	5
⋮	⋮	⋮
24	48	2

We could draw a line, since points between the points gave pairs of ages in years and months, and even days. We were not concerned, however, with these points. Instead, we discussed the reasons why this particular information gave us a straight line and not just a row of points as in the other problem. This seemed quite understandable to the children.

We returned to the two columns of figures on the board representing the ages of Tommy and his father over a period of years. The children were asked to look at these a bit differently in attempting to answer the question, "When Tommy was one year old and his father twenty-five years old, his father was how many times as old as Tommy?" The answer came quickly, and we proceeded to answer this question for each pair of numbers on the board and to write these "how many times as old" numbers in a third column opposite the other two columns. It was amazing to the pupils how very rapidly the numbers in that third column became smaller, until we found that when Tommy became 24 his

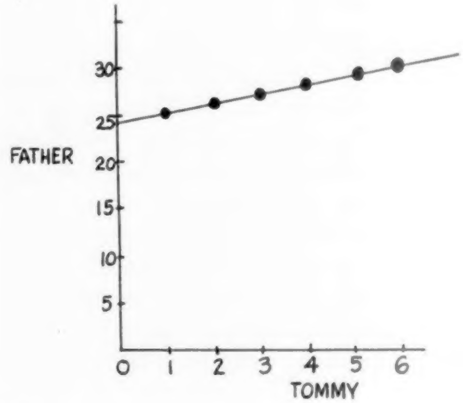


Figure 3

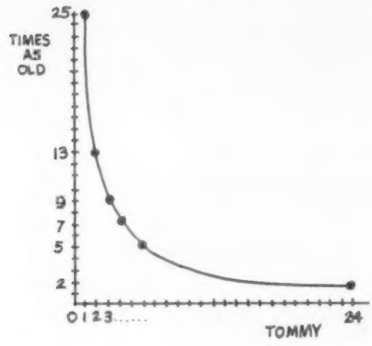


Figure 4

father would be only twice as old as Tommy! And if we had started to compare the ages when Tommy was but a few hours or a day or two old—what a very large number we could have had! To make a graph of this new information we decided to use the x-axis for Tommy's age(s); and on the y-axis we wrote the "how many times as old" numbers beginning at the zero point and using a unit of 5, remembering that this is not five years, but "five times as old," etc. Again, pupils volunteered to go to the board and plot the points representing pairs of numbers: 1 year old, 25 times as old; 2 years old, 13 times as old; 5 years old, $4\frac{4}{5}$ times as old, etc. Soon we could begin to see that in this graph we were not plotting a straight line. (See Figure 4.) After quite a number of points had been placed on the graph, someone offered to try to draw as smooth a curve as possible through the points. All were delighted to learn that this was a part of the curve called a "hyperbola," "a word you really shouldn't be expected to know at least until you have high school mathematics."

Interest was at such a peak by now that it seemed safe to try one more "pattern." The formula for the area of a square was used. Since the fourth grade had not had the words "formula" and "area," the work was most informal. A small square was drawn on the board and the side called "1." A second square was drawn and the side called "2." The children were then asked, "How many of the small squares could be put inside the square whose side is "2"? Very quickly someone was sure he could draw four such squares in the new figure, and he went to the board to show how it could be done. In a similar way we did a square whose side was "3"; here there was discussion as to whether it would contain 6 or 9 of the smaller squares, but very soon a pupil went to the board and drew in the 9 squares. We repeated the experiment with a "4" square, and then stopped to examine the "pattern" that was being developed. Without further drawings the children could see that the "5" square would have 25 small squares, and that in the "6" square there would be 36 small squares. We now arranged these numbers in pairs in two columns on the board and prepared to make our graph, using one axis for the "sides" and the other for the "insides." As points were placed on the graph representing the pairs of numbers (Figure 5), we could see that a still different kind of graph was developing, and when the "smooth curve" was drawn by a pupil, all again were delighted to know that they had a graph of part of a "parabola."

Enthusiasm was high. There was little time left for further exploration. We

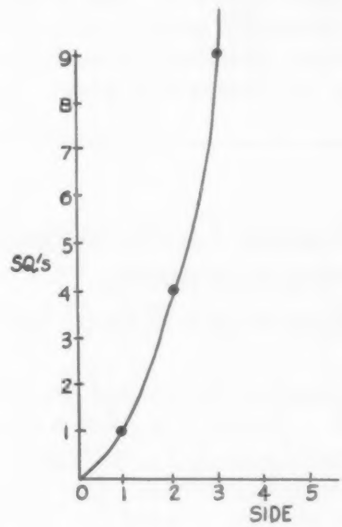


Figure 5

talked of information we might "read" from our final graph—information which we did not "plot" in making the graph—and of the kinds of graphs other sets of ordered pairs might give us. For example, that afternoon the children were to attend a play. The tickets were 15¢ each. We discussed how we might plan a graph to include the price of any number of tickets. When asked, "Do you think the points of this graph would lie in a straight line or along a curved line?" one boy quickly said, "A curve"; but a more discerning youngster said, "No, I'm almost sure it would be a straight line, because the number of tickets and the price will be changing at the same rate." Then they made a graph as in Figure 6.

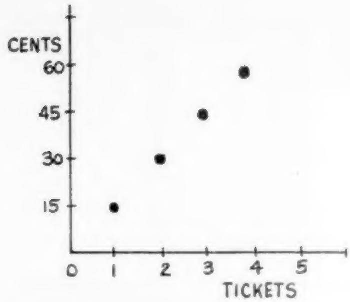


Figure 6

Reflections

Of what value is such an experiment? It is difficult to say. One outcome was the realization by all the pupils that, in problems involving pairs of numbers which have a relationship, some type of graph is possible; that the type of graph is dependent upon the kind of relationship existing between the numbers; and that the graph can give information in addition to that which was used in plotting some points.

The experiment was interesting and the results most satisfying. Elementary pupils are undoubtedly capable of understanding deeper mathematical concepts than usually are expected of them. Yet, although high in motivation, one is not convinced that an activity such as this should be introduced as a part of the "basic curriculum" in fourth grade; if it is used, it should be a natural outgrowth of the basic content. Surely, however, this experiment is an illustration of the type of work which might be provided as enrichment material for the more able pupils at the middle-grade level. Too, teachers are challenged every time they have an opportunity to lead young children to make interesting and challenging discoveries for themselves in the field of mathematics. Such experiences at any level are delightful for both pupils and teachers. And teachers need to be alert to recognize the opportunities for going deeper with pupils while at the same time aiding the "action research" which will surely help to provide some of the answers we are seeking regarding "concepts," "content," "grade placement," etc., in the field of mathematics.

Eunice Lewis appointed vice-president, secondary school level

The Board of Directors of the National Council of Teachers of Mathematics has appointed Eunice Lewis to the position of vice-president, secondary school level, to fill the vacancy created by the recent resignation of William Glenn. Miss Lewis is associate professor of education at the University of Oklahoma, teaching mathematics classes in the University High School. In addition, Miss Lewis supervises the mathematics teacher trainees in the university.

Remainders in division and a floor number line

Here are a number of ideas to introduce in your own classroom. They should prove both meaningful and thought provoking.

Remainders in division

Use a clock model or drawing to help your children with the concept of remainders in division. Provide a clock for each of the numbers you are using as divisors. The clock for 3's will have three points, the clock for 4's will have four points, the clock for 5's, five points, and so on.

Give children a series of numbers and have them find at what point on the clock for the 4's each of the numbers lies. If the remainder is 3, the number will be placed on the outside of the circle in the space opposite the point for the number 3. Children can draw their own clocks and insert the numbers in the proper places as they determine the remainders. Figure 1 shows the clock and the set of numbers as presented to the children. Figure 2 shows the completed work.

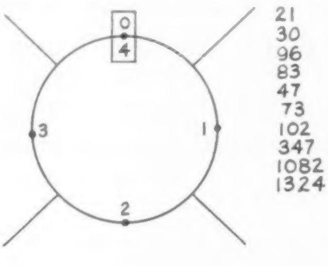


Figure 1

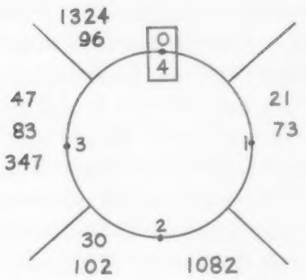


Figure 2

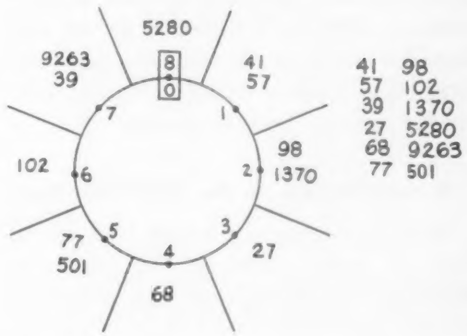


Figure 3

The same procedure can be followed for any of the numbers through 9. Figure 3 illustrates where each of the numbers beside the figure would fall. For example, the remainder for $68 \div 8$ is 4; therefore, 64 is placed by point 4 on the clock.

As children work with these clocks, they see that the number 4, when used as a divisor, can only have remainders of 1 through 3; that a divisor of 8 can only

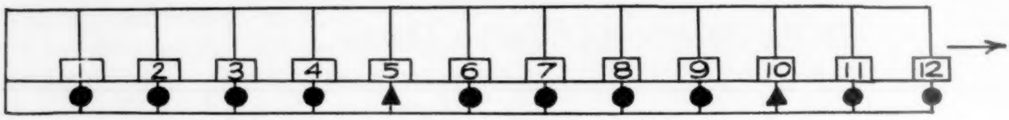


Figure 4

have remainders of 1 through 7. Thus children are led to the generalization that possible remainders are always one less than the divisor.

Work with remainders may be begun at the time the multiplication and division facts are being learned. At the appropriate time, this work can be extended to 3-, 4-, and 5-place numbers. When children work with the larger numbers, they can be encouraged to look for patterns as they study the numbers in each position around the clock. They may be able to discover a way to tell if a number is evenly divisible by four before dividing. They will see interesting patterns of odd and even numbers as they analyze the numbers beside each remainder around the circle.

When it is desirable to have many numbers beside each remainder, you may wish to divide the class, letting each group of children work on a different set of numbers. All numbers can then be placed on a large class chart or on a chalkboard drawing for analysis and discussion.

A number line on the classroom floor

You can make a number line on the floor of your classroom by stretching masking tape along one wall. Place the number line about two feet from the wall to allow easy movement by children. If your floor is covered with tile squares, the length of a tile can be the unit of the number line. Mark units with points and numbers, as in Figure 4. Of course, a number line can be constructed on any floor by marking and labeling equal segments along the line.

Children face the number line and take steps to the right or to the left. The following understandings can be developed by using such a number line.

1. As you take steps to the right, the numbers become larger; as you take steps to the left from any given point, the numbers become smaller.
2. When you take two steps at a time, you are counting by 2's. You can count by 2's beginning with any number Counting by 2's requires skipping a step or skipping a number.
3. Two numbers may be added by starting with one number and stepping off the other.

   $3+2=\square$

   Start at 3; move 2 steps to the right.

   $3+2=5$

   $8+6=\square$

   Start at 8; move 6 steps to the right.

   $8+6=14$

4. The order in which numbers are added does not change the answer. Take the examples above. Start with 2 and step off 3; start with 6 and step off 8. Children will see that they get the same answer regardless of which addend is first.
5. One number may be subtracted from another by starting with the total and moving to the left the number of steps indicated by the known addend.

   $6-4=\square$

   Start at 6; move 4 steps to the left.

   $6-4=2$

   $14-5=\square$

   Start at 14; move 5 steps to the left.

   $14-5=9$

6. The missing number may be found by starting with the known addend and

finding how many steps it takes to reach the total.

$6+\square=11$

Start at 6; see how many steps are needed to reach 11.
This gives the missing number.

$6+5=11$

7. Two numbers can be compared by matching the known part with the total and finding how many units do not match.

   Six is how many more than 4?
   Find 4; find 6; 4 of the 6 units will match.

   Two of the 6 units will not match.
   Six is 2 more than 4; or 4 is 2 less than 6.

   What is the difference between 16 and 9?
   Find 16; find 9.
   Nine matches with 9 units of the 16.
   Seven of the 16 units are left after the matching.
   So 16 is 7 more than 9; or 9 is 7 less than 16.

8. Use the number line to help children analyze problem-solving situations.

   Only 8 of our 13 girls are at school today.
   How many are absent?
   Start at 8; show 13; find how many are needed with 8 to make 13.
   Write the number sentence: $8+5=13$.

   There were 8 cookies in the box. We ate 6 of them. How many were left? Start at 8; move 6 steps to the left.
   Write the number sentence: $8-6=2$.

   Jack bought three 4¢ stamps. How much money did he pay for them? Move by 4's to the right. Make 3 moves.
   Write the number sentence: Three 4's = 12; or $3\times4=12$.

   Joe has 4 red cars, 2 blue cars, and 3 green cars. How many cars has he? Start at 4; move 2 steps to the right to 6; move 3 more steps to the right to 9.
   Write the number sentence: $4+2+3=9$.

   Of course children should share in the analysis of the number situation to determine whether moves should be made to the right or to the left to find the answer.

9. Children may be encouraged to develop games which can be played with partners on the number line. One such game follows.

If the number line is 25 units long, the game may be called "The Game of 25." Partners draw grouping cards to determine the number of moves to make along the line.

Sally draws the left card shown in Figure 5, makes four moves, and places a marker at 4 on the number line. Her partner draws the other card shown in Figure 5, makes 6 moves, and places her marker at 6 on the number line. They continue to draw cards and to make moves as indicated by groups on the cards. The person who reaches 25 first is the winner.

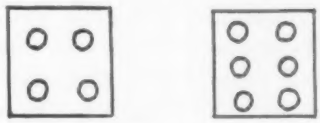


Figure 5

The game may be played beginning with 25 and moving to the left. The person to reach the beginning of the number line first is the winner.

For another appealing game, design arrow cards for children to draw. Arrows indicate moves to right and left. Each child draws a card and makes his moves. The winner is the child who first reaches 25.

The top card in Figure 6 is interpreted as follows: move 3 steps to the right; move 2 two steps to the left. The lower card in Figure 6 is read: move 2 steps to the left; move 5 steps to the right.

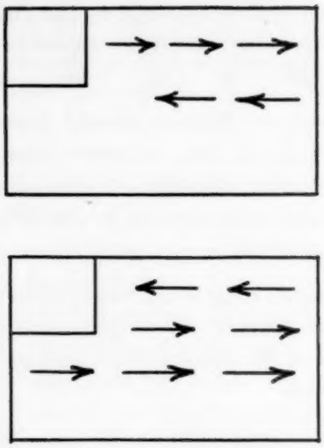


Figure 6

Children soon learn to save moves as they see that the same number of arrows to the right and to the left cancel each other out, leaving them at the same number. They can make only the moves indicated by the difference between the two sets of arrows.

Many children are quite creative in thinking up games of this type to play. As children practice these games, they begin to develop the ability to move by 2's and 3's instead of by 1's. Some children become adept at thinking of the moves abstractly without taking steps at all. Such growth toward abstract thinking should be encouraged.

Games and activities of the type described for the floor number line may start at kindergarten level and may increase in difficulty as long as they afford interest and motivation for practice or provide a means of new learning. Adaptations may be made to include simple multiplication, division, and fractions.

Those final digits

This note is an addendum to Catherine Stern's interesting article in THE ARITHMETIC TEACHER of December, 1960. For a number of years, I have had students test the likeness of final digits in the products of complimentary numbers for various bases: 7 and 3, 6 and 4, 8 and 2 for base ten; 5 and 3, 6 and 2, 4 and 4 for base eight; 7 and 5, 9 and 3, 8 and 4 for base twelve; and similarly for other bases.

For most students this proved to be just an interesting trick. A few wanted to know the principle behind the trick and how the principle might be proved. The following proof was developed:

Let N be the number base, and let a and $N-a$ be the complimentary numbers whose products we compare. Let k be any integer such that $1 \leq k \leq N-1$.

The first row below shows the products of a in ascending order. The kth term is ka. The second row shows the products of $N-a$ in descending order. The kth term is $(N-k)(N-a)$.

$$\begin{array}{ccc} a & 2a & \cdots \\ (N-1)(N-a) & (N-2)(N-a) & \cdots \\ & ka & \cdots \\ & (N-k)(N-a) & \cdots \end{array}$$

In the first row, the final digit in the kth product is the final digit in the number ka. In the second row, the final digit in the kth product is the final digit in the number $(N-k)(N-a) = N^2 - N(k+a) + ka = 100 - 10(k+a) + ka$. Since neither 100 nor $10(k+a)$ can influence the final digit of the number, the final digit of this product must be the final digit of the number ka.

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Enrichment for the talented in arithmetic: a local program for grades 4, 5, and 6

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Editor's introduction

Doctor Rudd is Associate Professor of Education at Tufts University and is serving as Arithmetic Consultant to the Newton (Massachusetts) Public Schools for the project he describes here. Professor Rudd will give a more detailed account of this project in connection with the program of the Thirty-ninth Annual Meeting of The National Council of Teachers of Mathematics to be held at the Conrad Hilton Hotel, Chicago, on April 5–8, 1961. (J.F.W.)

Great strides have been taken recently in the development of new curriculum materials in mathematics. That these changes are long overdue is recognized generally. It is further recognized that most of the impetus for these changes has been furnished by foundations which have generously supported a wide variety of projects. An area of growing concern is that of the responsibilities of local school systems in the development and use of new curriculum materials. This area is important for at least two reasons: one, it seems highly unlikely that the present magnitude of support from other sources will continue indefinitely, and two, the processes of self-development, adaptation to individual needs, and self-appraisal are extremely vital in curriculum improvement.

Questions basic to setting up a special public school program

A public school system which is concerned with the development and implementation of curriculum materials must assume that this venture will be both expensive and time consuming. Furthermore, several basic questions must be answered. Six of these questions follow.

1. What are the initial steps in such a program?
2. How and by whom should the new materials be prepared?
3. What is the best solution to an orientation of the teachers to the new materials?
4. How are pupils to be screened for the program?
5. What procedures are to be used in presenting these new materials to children?
6. What are the steps to be taken in regard to public relations?

These six questions have been answered in part by the efforts of the public schools

of Newton, Massachusetts, in the development during the past two years of a special program for the more able pupils in grades four, five, and six, in a program which has been labeled "Arithmetic for the Academically Talented" (ATA).

The initial steps taken in this program involved a special committee formed in the fall of 1958 whose purpose was to explore the needs of the more able pupils in grades four, five, and six, and to make recommendations regarding the meeting of these needs. This committee recommended that the curriculum area of arithmetic should be examined with a view toward a special program for talented pupils. Consequently, a workshop group of twenty-two teachers and principals from ten elementary schools held a series of eight meetings, the last six under the leadership of an arithmetic consultant. The utilization of group-process techniques in both the committee and the workshop proved to be valuable in setting the tone for further development of this special program.

The development of new materials and workshop programs

Curriculum materials were developed by three members of the workshop group: two sixth-grade teachers and the arithmetic consultant. These three persons worked for two summers in the development of units of work in six areas of arithmetic instruction:

Historical development of numbers
Laws of numbers
Measurement
Other scales of notation
Primes and factors
An introduction to sets.

Units in the first four areas were developed during the summer of 1959 and were used in the classrooms in 1959–1960. The remaining units were developed in the summer of 1960. In addition to these units, many materials for both teachers and pupils have been obtained. Among these are professional books for teachers, subscriptions to THE ARITHMETIC TEACHER, enrichment books and pamphlets, films and filmstrips, number frames and abaci, and measurement equipment.

The mathematics background of elementary-school teachers is very important. This prerequisite to teaching arithmetic is of even greater importance in a special program of enrichment for talented pupils. Consequently, teacher workshops have been conducted with all of the teachers prior to their work with children. At the present time, five workshops for teachers in the ATA program have been conducted over a two-year period. These workshops have been under the direction of an arithmetic consultant; they have been given in the public schools, and the participating teachers received one in-service credit. The workshop sessions have been devoted to teacher orientation to the new materials and to discussion of techniques for screening the pupils and administration of the program. The minimum time devoted to each workshop has been fifteen hours, spread over a series of ten weekly or biweekly sessions.

A great deal of care is employed in the screening of pupils for the program. Five screening criteria are used and equal emphasis is given to each. These criteria are:

1. Intelligence quotient
2. Achievement test score
3. Score on a noncomputational arithmetic test
4. Classroom progress in arithmetic
5. Observation check list filled out by the classroom teacher on twelve characteristics indicative of talent in the area of arithmetic.

It is the judgment of both teachers and principals that the use of these five criteria has been effective in screening children for the program. In many cases, the arithmetic consultant has assisted teachers and principals in making final decisions regarding the selection of the pupils.

Putting the program into effect

The work with children has been initiated in each school after the teacher has finished the workshop and the children have been selected. In some schools, the teachers have handled the work on a self-contained classroom basis with the talented pupils forming their top arithmetic group. In other schools, the pupils from more than one classroom have been grouped on a departmentalized basis. Both procedures have been received with enthusiasm by teachers and principals. Whether one plan is better than the other for a particular school depends to a large extent upon such factors as preparation of teachers, number of classrooms per grade, number of talented pupils, and the general administrative arrangement. The arithmetic consultant through conferences, classroom visits, and demonstration teaching, has assisted with the implementation of the program in the classrooms.

The program has been initiated in the elementary schools without the fanfare of extensive publicity. As questions have arisen, they have been answered both in conferences and in explanations of the program at PTA meetings. Also, articles summarizing the program have been published in the PTA newsletter and the annual school report. It is felt that these steps in public relations have been adequate. The program has been well received by teachers, pupils, and parents.

Beginning with the initial workshop group of representative teachers from ten elementary schools, the program has expanded, first to other grades in the ten schools and subsequently to the remaining elementary schools. The incidence of talented pupils is higher in some schools than in others; however, one or more teachers in each of the twenty-five elementary schools in Newton are now participating in the program. Quantities of pupil materials and teacher guides have been printed, and it is expected that the program will continue to grow as a basic part of arithmetic instruction in Newton.

Experiments in mathematics

The Board of Directors of the National Council of Teachers of Mathematics appointed a committee to gather information and to analyze experimental mathematics programs. This committee would like your help in identifying such programs. It has limited its analysis to programs that (1) have printed instructional materials other than courses of study, (2) are of at least one semester in duration, and (3) are not sponsored by a commercial publishing firm. The committee will examine all material, ask the experimenter to make a short personal report, and publish as many of the reports as possible.

If you know of experimental programs or have one of your own, we would appreciate receiving your material or having you contact the chairman, whose address is:

PHILIP PEAK, Chairman
Committee on the Analysis
of Experimental Programs
School of Education
Indiana University
Bloomington, Indiana

Books and materials

Magic House of Numbers, Irving Adler. New York: The John Day Company, 1957. Cloth, 128 pp., $3.00.

Numbers Old and New, Irving and Ruth Adler. New York: The John Day Company, 1960. Cloth, 48 pp., $2.00.

The author points out that numbers are tools with which we work. He also claims that numbers are toys with which we can play. He then proceeds to show what he means by the latter statement as he challenges the reader with number puzzles, games, curiosities, and a few card tricks of the mathematical type. But the purpose of the books is not just to amuse and challenge. The materials are chosen to help develop a meaningful background in mathematics.

Looking first at *Magic House of Numbers,* we note that the reader is introduced to "patterns" or "steps" within number sequences. The search for these patterns sets the mood for the other creative activities to follow. Considerable time is spent discussing numeration systems to bases ten, eight, twelve, and two. The binary system is then used to explain such games as Nim, Tower of Hanoi, and Chinese rings, as well as the flip-flop circuits of modern electronic computers. Short, challenging puzzles, some with numbers and some without numbers, are well chosen for interest and worthwhileness in terms of quantitative thinking. Answers are grouped together at the end.

The *Magic House of Numbers* can be used individually or in groups with those who have a good background in arithmetic. A teacher will appreciate the explanations of such practices as "casting out 9's" and the use of different number bases. The symbolism is simple and most of the explanations are followed easily. The simple diagrams add interest and meaning.

Numbers Old and New is a little book perhaps for pupils below the seventh grade. It gives a taste of many of the ideas found in *Magic House of Numbers.* Some topics included are numeration systems; number lore as developed by the Greeks, including such types of numbers as triangle, square, perfect, odd, and even numbers; different algorisms that have been used for multiplication; and number tricks.

The presentation is straightforward. The definitions of ideas are given with illustrations to clarify the meaning. The accompanying drawings are well chosen and attractive. The materials in this book will add greatly to the enjoyment and understanding of the background of arithmetic. This presentation is not as challenging to the reader as is that in the *Magic House of Numbers.*

The authors have carefully selected historical materials to help pupils understand the arithmetic they use. Old puzzles have sometimes been clothed in new settings to lend more interest. Care has been taken to present correct mathematical information.

The authors are to be commended on the preparation of these timely books as an aid in upgrading the interest and achievement in mathematics.

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Using the Language of Algebra in Arithmetic, William H. Glenn, William G. Mehl, Dean S. Rasmussen. 847 North

E. Street, San Bernardino, California: Franklin Teaching Aids, Inc., 1960. Paper, pupil edition, 119 pp., $2.00; teacher edition, 59 pp., $2.00.

As the authors imply in the title, this publication provides material which introduces the language of algebra into arithmetic. The twenty-two lessons presented are designed to be used as supplemental to a regular text. While they are primarily for use on the seventh- and eighth-grade level, the authors also indicate that advanced fifth- and sixth-graders may wish to make use of the material. To the reviewer, a third use occurs: perhaps ninth-grade teachers might use these lessons to supplement work either in algebra or general mathematics.

The preface in the Teacher's Edition expresses the general purpose of the lessons and the plan for accomplishing this purpose: "The purpose of this material is to develop the means by which mathematical relationships can be expressed in very simple mathematical form. The combination of number symbols and operation symbols forms the structure through which this is accomplished. The result is referred to as 'the language of algebra,' since each mathematical expression represents a set of directions or a mathematical sentence. An understanding of this structure causes mathematics to become a powerful and useful tool to express ideas."

Lessons include operations with positive integers, fractions, and mixed numbers. The usual symbols of operation are used, including the parentheses, the bar, brackets, and braces. Attention is given to order of operations. One lesson is entitled "Numbers Have Many Names." Writing mathematical sentences in horizontal form is stressed. Ratio is presented as a comparison by division, proportion as two equal ratios, and percentage in terms of a proportion. Parenthetical expressions and their applications in formulas make up three lessons. Exponents, powers, factors, prime factors, and divisibility are included. One lesson is presented on the "ten-ness of our number system." The last lesson is a matching test covering the ideas which have been considered in the first twenty-one lessons.

The authors state that "each lesson should be looked upon as a means of developing greater facility in the use of mathematical symbolism." "Word problems," they say, "then can be analyzed and solved by first writing the solution in indicated operation form." To develop this facility, each lesson in the Pupil Edition presents several examples, and their solutions, illustrating the particular objective of the lesson and the method to be used. Following these statements, practice material is provided. Pages are perforated for easy tearing out if the teacher wishes to use only particular pages.

Leafing through the Pupil Edition, the reviewer gets the feeling that this is a drill book in which the children merely follow the mechanical motions indicated by illustrations. And it *could* be used in just this way. However, if oral discussion is conducted (as the Teacher Edition directs) children can be guided into understanding and into discovering for themselves many of the properties of mathematics.

For the teacher who is unaccustomed to having children use (consciously) the commutative and associative properties of addition and multiplication, and identity elements in addition and multiplication, comment in the Teacher Edition for Lesson I will provide a source of security. Answers are written out in detail for all lessons; these too will bolster the mathematical morale of the teacher who has felt somewhat inadequate with respect to symbolism and the expression of ideas.

The reviewer feels that many teachers in grades five through eight will welcome this publication as a source through which they may extend the mathematics presented in the usual arithmetic text.

ELINOR B. FLAGG
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Books received

Exploring Mathematics on Your Own, series of seven pamphlets with five in preparation, Donovan A. Johnson and William H. Glenn. St. Louis: Webster Publishing Company, 1960. Set of seven, $4.00. (Pamphlet titles: *Sets, Sentences, and Operations; The Pythagorean Theorem; Topology; Understanding Numeration Systems; Fun with Mathematics; Number Patterns; Invitation to Mathematics.*)

My Number Friends, Pupils and Teachers Editions for Grades 1 and 2, Ruth L. Cole and Harry Karstens. Chicago: Lyons and Carnahan, 1959. Grade 1, 128 pp. $.96. Grade 2, 160 pp., $1.20.

Ready to Begin Numbers, Pupils' Book 1 and Book 2, Elda L. Merton and Leo J. Brueckner. Philadelphia: The John C. Winston Co., 1960. 160 pp. each, $1.04.

Seeing Through Arithmetic Tests for Grades 3, 4, 5, and 6. Maurice L. Hartung, Henry Van Engen, E. Glenadine Gibb, and Lois Knowles. Chicago: Scott, Foresman and Company, 1960. Specimen set, $1.00.

Professional dates

The information below gives the name, date, and place of meeting with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE ARITHMETIC TEACHER. Announcements for publication should be sent at least ten weeks early to the Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.

NCTM convention dates

Thirty-ninth Annual Meeting

April 5–8, 1961
Conrad Hilton Hotel, Chicago, Illinois
Hobert Sistler, Morton High School West, 2400 Home Avenue, Berwyn, Illinois

Joint Meeting with NEA

June 28, 1961
Atlantic City, New Jersey
M. H. Ahrendt, 1201 Sixteenth Street, N.W., Washington 6, D.C.

Twenty-first Summer Meeting

August 21–23, 1961
University of Toronto, Toronto, Canada
Father John C. Egsgard, St. Michael's College School, 1515 Bathurst Street, Toronto 10, Canada

Sectional meetings of the Illinois Council of Teachers of Mathematics

There will be six sectional meetings of the Illinois Council of Teachers of Mathematics in March and April, 1961. Themes of these meetings include: "Function of Modern Mathematics," "Meeting Today's Challenges in Mathematics," and "Recent Developments in Teaching of Mathematics." These meetings are specifically designed for teachers at the elementary, secondary, and college levels. The dates, places, and chairmen of the meetings are:

March 25: Monticello College, Godfrey, Ill.; Evelyn Trennt

April 1: Southern Illinois University, Carbondale; W. C. McDaniel

April 14: Eastern Illinois University, Charleston; Alphonso DePietro

April 15: Western Illinois University, Macomb; Jerry Shyrock

April 22: Illinois State Normal University, Normal; Hal Gilmore

April 29: Sterling Township High School, Sterling; Charles E. Schulz and Chester Sherman

For detailed information please write to the chairman of the specific meeting.

For general information, write to: T. E. Rine, Chairman of Public Relations, Illinois Council of Teachers of Mathematics, Illinois State Normal University, Normal, Illinois

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